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The compatibility heuristic in non-categorical hypothetical reasoning: Inferences between conditionals and disjunctions



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ABSTRACT

A new theory explains how people make hypothetical inferences from a premise consistent with several alternatives to a conclusion consistent with several alternatives. The key proposal is that people rely on a heuristic that identifies compatible possibilities. It is tested in 7 experiments that examine inferences between conditionals and disjunctions. Participants accepted inferences between conditionals and inclusive disjunctions when a compatible possibility was immediately available, in their binary judgments that a conclusion followed or not (Experiment 1a) and ternary judgments that included it was not possible to know (Experiment 1b). The compatibility effect was amplified when compatible possibilities were more readily available, e.g., for 'A only if B' conditionals (Experiment 2). It was eliminated when compatible possibilities were not available, e.g., for 'if and only if A B' bi-conditionals and exclusive disjunctions (Experiment 3). The compatibility heuristic occurs even for inferences based on implicit negation e.g., 'A or B, therefore if C D' (Experiment 4), and between universals 'All A's are B's' and disjunctions (Experiment 5a) and universals and conditionals (Experiment 5b). The implications of the results for alternative theories of the cognitive processes underlying hypothetical deductions are discussed.

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1. Introduction

When Obama warned Congress to accept his proposals about tax cuts or face the ad hoc cuts of the sequester, was it accurate to infer at the time that if Congress accepted his proposals, the sequester would *not* be introduced? Hypothetical inferences of this sort are made without recourse to factual information – at the time it was unknown whether Congress would accept Obama's proposals, and it was unknown whether the sequester would be introduced. They contain no categorical information, and so they require a reasoner to infer a conclusion that is consistent with several alternatives, from a premise that is consistent with several alternatives.

Inferences between conditionals and disjunctions allow a consideration of alternatives in the absence of categorical information that is crucial for genuinely hypothetical prediction (e.g., [Wason & Johnson-Laird, 1972](#)). Despite their importance, the past 50 years of research has focused on conditionals and disjunctions separately. It has examined hypothetical inferences anchored by a categorical assertion of fact, e.g., 'If Congress accepts Obama's proposals, the sequester will not be introduced. The sequester was introduced. Therefore, Congress did not accept his proposals.' (e.g., [Evans, 1983](#); [Johnson-Laird & Tagart, 1969](#); [Newstead, Griggs, & Chrostowski, 1984](#); [Roberge, 1976, 1977](#); see [Manktelow, 2012](#) for a review). Almost all research on hypothetical inference has focused on such 'after-the-fact' hypothetical inferences. But in everyday life people also reason in situations in which the facts are not known.

From the instruction: 'Release the clutch gently or the car will stall', does the conclusion 'If I don't release the clutch gently, the car will stall' follow validly? Logicians disagree about whether it should (e.g., [Kripke, 1959](#); [Quine, 1974](#)), and psychologists dispute whether, for most logically naïve individuals, it would (e.g., [Evans & Over, 2004](#); [Johnson-Laird & Byrne, 2002](#)). Do people make these inferences? The answer remains largely unknown. We report a series of 7 experiments that reveal people's reliance on heuristic processes to do so.

1.1. Inferences between conditionals and disjunctions

Only five empirical observations have been made about how people make inferences between conditionals and disjunctions: Given a claim, e.g. 'there's an Ace in the hand or there's a King', participants rate as highly convincing the conclusion, 'if there's not an Ace, there's a King' ([Johnson-Laird & Byrne, 2002](#); [Over, Evans, & Elqayam, 2010](#)). They generate paraphrases of 'A or B' using 'if' readily, more so than they evaluate conclusions based on 'if' ([Ormerod & Richardson, 2003](#); [Richardson & Ormerod, 1997](#)). However, given multiple premises, e.g., 'A or B. If B, not not-C. Therefore if not-A, C', they spontaneously make intermediate inferences such as 'if B, C' but less often, 'if not-A, B' ([Braine et al., 1995](#); see also [Braine, Reiser, & Rumin, 1984](#); [Osherson, 1975](#); [Rips, 1983](#)). From a sequence of conditionals, e.g., 'if A, B, if B, C' they infer a conditional conclusion, 'if A, C' ([Byrne, 1989a](#); [Santamaria, Garcia-Madruga, & Johnson-Laird, 1998](#)), but from a sequence of disjunctions, e.g., 'either A or else B, either B or else C', they find it difficult to make an inference ([Johnson-Laird, Byrne, & Schaeken, 1992](#)). And they accept more inferences that contain an affirmative disjunction 'A or B, therefore If not-A, B', compared to a negated disjunction 'Not-A or B, therefore If A, B' ([Oberauer, Geiger, & Fischer, 2011](#)).

But these observations have been based on only a few of the possible inference forms between conditionals and disjunctions, and nothing is known about how people make most of the 16 distinct inferences that can be constructed from a conditional of one of four polarities (if A B, if A not-B, if not-A B, if not-A not-B) to a disjunction of one of four polarities (A or B, A or not-B, not-A or B, not-A or not-B), and the corresponding set of 16 inferences from a disjunction to a conditional. Our aim is to provide a systematic and comprehensive examination of the complete set of 32 inferences between disjunctions and 'if A B' conditionals, as well 'A only if B' conditionals, 'if and only if A B' bi-conditionals, and 'All A's are B's' universals, including both inclusive disjunctions, 'A or B or both' and exclusive disjunctions 'A or B but not both'. We propose a new theory for inferences between conditionals and disjunctions based on a heuristic that identifies compatible possibilities. No theory is available that encompasses every sort of hypothetical inference: the scope of ours is to provide a fragment of a new account for

'non-categorical' hypothetical inferences based on a short-cut adaptation of an algorithm for 'after-the-fact' ones.

2. The compatibility heuristic

At the heart of our proposal is the idea that people take a short-cut when they make inferences between a conditional and a disjunction. They identify a single compatible possibility by a heuristic process that has three characteristics: (a) it simulates initial possibilities for the conditional and disjunction, (b) it seeks to identify a compatible possibility between the initial possibilities for the conditional and disjunction, and (c) if it cannot do so, it accesses the most immediately available alternative. It is based on the idea that people reason by envisaging possibilities, and we sketch this idea first.

2.1. Possibilities

Logicians have proposed that the assertability of a conditional is provided by a possible worlds semantics, in which, 'a possible world is the ontological analogue of a stock of hypothetical beliefs' (Stalnaker, 1968, p.102). The semantics is based on modal logic assumptions about the relations of necessity and possibility (Kripke, 1959), e.g., 'it is possible that A' is defined as, 'it is not the case that it is necessary that not-A'. Psychologists have proposed that people understand a conditional e.g., 'if there is a lion in the zoo, there is a giraffe' (if A B) by envisaging the *possibilities* with which it is consistent, rather than its truth. They think about a possibility corresponding to the way the world would be if the assertion were true, e.g., 'there is a lion and there is a giraffe' (e.g., Johnson-Laird & Byrne, 2002):

lion giraffe

For clarity, the possibility can be captured in a diagram devoid of content:

A B

but it is proposed that people construct iconic 'mental models' that capture aspects of the elements simulated (e.g., Johnson-Laird, 2006). Their interpretation differs from a conjunction because they make a 'mental note' that there may be alternatives, which they have not yet thought about (Johnson-Laird & Byrne, 1991). The possibilities are guided by a principle of parsimony, because of the constraints of working memory (Barrouillet, Gauffroy, & Lecas, 2008; Espino & Byrne, 2012; Goodwin & Johnson-Laird, 2005; Johnson-Laird et al., 1992). The initial possibility contains the elements mentioned in the conditional.

Likewise, when people understand a disjunction 'there is a lion in the zoo or there is a giraffe' they envisage an initial possibility in which 'there is a lion', and as an alternative another possibility in which 'there is a giraffe' (Johnson-Laird, Lotstein, & Byrne, 2012; Khemlani & Johnson-Laird, 2009):

A
B

and separate possibilities are captured in the diagram by separate lines. The initial possibilities are incomplete because of the principle of parsimony, and so people initially envisage 'there is a lion', without thinking about whether there is a giraffe in that situation.

2.1.1. Alternative possibilities

The possibilities people consider are also guided by a principle of truth and so they do not imagine the false possibility ruled out by the conditional, 'there is a lion and there is no giraffe' (A and not-B) (Espino, Santamaria, & Byrne, 2009; Johnson-Laird & Byrne, 2002). Thus the possibilities they think

about differ from entries in a logical ‘truth table’ (Wittgenstein, 1953). Instead, they imagine an alternative true possibility, e.g., ‘there is no lion and there is no giraffe’:

A	B
Not-A	not-B

The alternative they envisage most readily depends on task demands, but the most immediately available alternative may correspond to the negation of each of the mentioned components (e.g. Jahn, Knauff, & Johnson-Laird, 2007).

Semantic and pragmatic factors modulate the possibilities people consider (Johnson-Laird & Byrne, 2002). Some people come to a conditional interpretation of ‘if’, they ‘flesh out’ a third possibility ‘there is no lion and there is a giraffe’, perhaps because they retrieve a counterexample, e.g., there is a tiger in the zoo as an alternative to a lion (not-A and B) (Byrne, 2005, 2007; De Neys, Schaeken, & d’Ydewalle, 2005; Markovits, Lortie Forgues, & Brunet, 2010):

A	B
Not-A	not-B
Not-A	B

Others come to a ‘bi-conditional’ interpretation, ‘if and only if there is a lion there is a giraffe’: they think of just the first two possibilities. And other conditionals are interpreted to refer to other relations, e.g., ‘if the product is advertised, sales increase’ is interpreted as an ‘enabling’ relation, consistent with ‘it is advertised and sales increase’ and ‘it is not advertised and sales do not increase’, and also with, ‘it is advertised and sales do not increase’ (A and not-B) (e.g., Frosch & Byrne, 2012; Goldvarg & Johnson-Laird, 2001). They retrieve a counterexample, e.g., the ads must be well placed (e.g. De Neys et al., 2005; Markovits et al., 2010):

A	B
Not-A	not-B
A	not-B

Likewise, for ‘there is a lion or there is a giraffe’ participants can think about the alternatives in more explicit detail, e.g., ‘there is a lion and there is no giraffe’, ‘there is no lion and there is a giraffe’:

A	not-B
Not-A	B

For an exclusive disjunction, ‘either there is a lion or else there is a giraffe but not both’ they consider only these two possibilities, whereas for an inclusive disjunction ‘there is a lion or there is a giraffe or both’, they also consider ‘there is a lion and there is a giraffe’:

A	not-B
Not-A	B
A	B

2.1.2. Inferences and possibilities

People make inferences based on the possibilities they have considered. Some inferences, e.g., *modus ponens* can be based on an inspection of the initial possibility. For ‘if there is a lion there is a giraffe’, they envisage initially ‘there is a lion and there is a giraffe’ (A and B), and because the possibility considered for *modus ponens*, ‘there is a lion’ (A) shares a common element with it, the conclusion ‘there is a giraffe’ (B) is endorsed (e.g., Johnson-Laird, 2006).

Other inferences require the construction and combination of alternative possibilities. The possibility considered for *modus tollens*, ‘there is not a giraffe’ (not-B) does not share a common element with the initial possibility envisaged for the conditional and so, many reasoners say that nothing follows (e.g., Johnson-Laird & Byrne, 1991). However, if they think about an alternative: ‘there is not a lion and there is not a giraffe’ (not-A and not-B), they identify a shared element:

<i>If A</i>	<i>B</i>	<i>not B</i>
A	B	not-B
Not-A	not-B	

The information in the categorical premise eliminates the inconsistent possibility, and they conclude, ‘there is not a lion’ (not-A).

An inference is never valid merely according to its form (Johnson-Laird & Byrne, 2002; p. 666), instead pragmatic and semantic modulation is crucial. Given ‘if Rachel is in Brazil she is not in Rio’ and the *modus tollens* information ‘Rachel is in Rio’, participants do not conclude ‘therefore she is not in Brazil’ because their geographical knowledge rules out the alternative, ‘Rachel is not in Brazil and she is in Rio’ (Johnson-Laird & Byrne, 2002). The counterexamples people retrieve can lead to the suppression of inferences, even of the simple *modus ponens* one (e.g., Byrne, 1989b; Byrne, Espino, & Santamaria, 1999). Hence, the conditional does not correspond to a truth-functional ‘material implication’ (Johnson-Laird & Byrne, 2002, p. 651) – it is based on *possibilities* rather than truth values, and content and context modulate the possibilities considered.

Unlike ‘after-the-fact’ hypothetical inferences such as *modus ponens* and *tollens* that contain categorical premises, inferences between conditionals and disjunctions are made in the absence of facts. Because they are not anchored in categorical information that would allow some alternatives to be eliminated, they place high demands on working memory and people may rely on short-cuts. The compatibility heuristic is based on the simulation of initial possibilities, and its inferential procedure is satisfied by the identification of a single compatible possibility, as we now outline.

2.2. The compatibility heuristic

Consider the steps that would need to be taken in a computer program designed to simulate the compatibility heuristic, given the following exchange: ‘Do I press the ‘laptop’ projector option or insert the connector into the laptop, or both?’ ‘Well, if you press the ‘laptop’ projector option, insert the connector into the laptop’ (A or B or both, if A B). The first step is to construct possibilities for the conditional and disjunction. A fully deliberative version of the program would construct the explicit set of possibilities:

<i>A or B</i>		<i>If A</i>	<i>B</i>
A	not-B	A	B
not-A	B	Not-A	not-B
A	B	Not-A	B

The short-cut version would construct only the initial possibilities:

<i>A or B</i>	<i>If A</i>	<i>B</i>
A	A	B
	B	
A	B	

The compatibility heuristic is based on an inferential procedure that compares the initial possibilities for conditionals and disjunctions.

The second step is to identify a compatible possibility. A program to simulate a deliberative process would carry out a systematic and exhaustive comparison of each of the explicit possibilities, by

executing both a compatibility and an incompatibility test. The compatibility test would ‘check off each possibility that is compatible with the premise and conclusion:

<i>A or B</i>			<i>If A B</i>		
A	not-B		A	B	✓
not-A	B	✓	Not-A	not-B	
A	B	✓	Not-A	B	✓

The incompatibility test would check whether there are possibilities compatible with the premise that are *not* compatible with the conclusion (in this case: ‘A and not-B’) and whether there are possibilities compatible with the conclusion that are *not* compatible with the premise (in this case: ‘not-A and not-B’):

<i>A or B</i>			<i>If A B</i>		
A	not-B	x	A	B	✓
not-A	B	✓	Not-A	not-B	x
A	B	✓	Not-A	B	✓

This version of the program would identify incompatible possibilities and judge the inference to be invalid. The inference is akin to a consistency judgment because it does not rest on a categorical premise that allows incompatible possibilities to be eliminated (e.g., [Johnson-Laird et al., 2012](#)).

A deliberative process consisting of a compatibility test and an incompatibility test of the fully explicit possibilities is likely to exceed working memory capacity. Another version of the program based on initial possibilities but with a similar compatibility and incompatibility procedure, would carry out a systematic comparison of the *initial* possibilities. The compatibility test would check-off each consistent possibility; the incompatibility test would check possibilities consistent with the premise that are not compatible with the conclusion, and vice versa (in this case there *are* some, albeit incomplete possibilities: ‘A’, ‘B’):

<i>A or B</i>		<i>If A B</i>	
A	x	A	B ✓
	B x		
A	B ✓		

Hence it would also judge the conclusion to be invalid. The incompatibility test checks there are no counterexamples, and it enables a judgment that a conclusion necessarily follows from the premise. But despite being based on initial possibilities, the full compatibility and incompatibility procedure may also exceed working memory capacity.

2.2.1. Compatible possibilities

We propose instead a heuristic process that is satisfied by the identification of a compatible possibility, an idea at the heart of the proposal that people reason by envisaging possibilities: ‘Reasoning depends on imagining the possibilities compatible with the premises’ ([Johnson-Laird, 2001](#), p. 441). People sometimes rely on short-cuts based on identifying a compatible possibility for inferences from conditionals, e.g., ‘When individuals carry out a series of inferences, they develop different strategies for coping with them ... Some reasoners consider the possibilities compatible with the premises’ ([Johnson-Laird & Byrne, 2002](#), p. 667), and for sentential inferences, e.g., ‘...drawing a single diagram that keeps track of all the possibilities compatible with the premises’ ([Van Der Henst, Yang, & Johnson-Laird, 2002](#), p. 434). They rely on identifying a compatible possibility when they make consistency judgments, e.g., ‘Reasoners appear to model the consistency of assertions by trying to envisage a compatible possibility’ ([Legrenzi, Girotto, & Johnson-Laird, 2003](#), p. 137). Such a short-cut seems feasible for inferences between conditional and disjunctive inferences e.g., ‘people try to construct a single set of mental models that is compatible with both statements’ ([Oberauer et al., 2011](#), pp. 116–117).

The compatibility heuristic is based on initial possibilities rather than explicit possibilities, and it is based on a procedure that tests compatibility only rather than one that tests both compatibility and incompatibility. We call it a ‘compatibility heuristic’ to reflect the long-standing emphasis on identifying ‘compatible’ possibilities (e.g. Johnson-Laird & Byrne, 2002), and to highlight its status as a heuristic derived from the fuller algorithm of considering possibilities. Different steps can be taken in a short-cut to identify compatible possibilities, and the particular steps will affect whether a compatible possibility is identified, and which compatible possibility is identified first. The first step is to compare the initial possibilities for the conditional and disjunction. Its second step is to test for compatibility only, checking off each possibility consistent with the premise that is also compatible with the conclusion:

<i>A or B</i>	<i>If A B</i>	
A	A B	✓
B		
A B		✓

It will lead to an erroneous endorsement of the conclusion in this case. The inference made by the heuristic is one of possibility rather than necessity (Barres & Johnson-Laird, 2003; Bell & Johnson-Laird, 1998).

Table 1

The consistent possibilities for 16 forms of inference between a conditional and inclusive disjunction used in Experiments 1a and 1b.

	Consistent Possibilities	If A B	If A not-B	If not-A B	If not-A not-B
Consistent Possibilities		A & B <i>Not-A & not-B</i> Not-A & B	A & not-B <i>Not-A & B</i> Not-A & not-B	Not-A & B <i>A & not-B</i> A & B	Not-A & not-B <i>A & B</i> A & not-B
A or B or both	A & not-B Not-A & B A & B	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: First Term Only	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Terms <i>Compatible Alternative</i>
A or not-B or both	A & B Not-A & not-B A & not-B	Shared Polarity: First Term Only	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: Neither Terms <i>Compatible Alternative</i>	Shared Polarity: Second Term Only Valid
Not-A or B or both	Not-A & not-B A & B Not-A & B	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Terms <i>Compatible Alternative</i>	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: First Term Only
Not-A or not-B or both	Not-A & B A & not-B Not-A & not-B	Shared Polarity: Neither Terms <i>Compatible Alternative</i>	Shared Polarity: Second Term Only Valid	Shared Polarity: First Term Only	Shared Polarity: Both Terms Compatible Initial Possibility

Key: The initial possibilities considered for a conditional and a disjunction are indicated in bold font, and the first alternative in italic font. The 4 inferences with a compatible initial possibility and the 4 inferences with a compatible alternative are indicated. The shared polarity of the terms in the premise and conclusion is provided: both terms (compatible initial possibility), neither term (compatible alternative), first term only, or second term only. The 4 valid inferences are also indicated.

For inferences between conditionals and inclusive disjunctions, it happens to be the case that compatible initial possibilities exist when the premise and conclusion share the polarity of both terms, as outlined in Table 1. It leads to the prediction that participants will endorse them. However, it is distinct from a superficial matching of linguistic referents, and it predicts that participants will endorse other inferences, even when the premise and the conclusion share the polarity of neither term, as we now outline.

2.2.2. Compatible alternatives

The compatibility heuristic engages a third step when it does not identify a compatible possibility. A computer program to simulate the compatibility heuristic given, 'If my keys are lost I'm locked out, therefore my keys aren't lost or I'm not locked out' would construct initial possibilities:

<i>If</i>	<i>A</i>	<i>B</i>	<i>not-A or not-B</i>
<i>A</i>	<i>B</i>	<i>not-A</i>	<i>not-B</i>
	<i>not-A</i>	<i>not-A</i>	<i>not-A</i>

The compatibility test identifies no possibility compatible with the premise and conclusion: some participants' immediate intuition may be that the inference is invalid. A computer program to simulate the compatibility heuristic could consider an immediately available alternative, based on the *opposite* of the elements mentioned in the conditional, i.e., 'my keys aren't lost and I'm not locked out'. The compatibility test could then identify a possibility consistent with the premise that is also compatible with the 'or both' possibility of the conclusion:

<i>If</i>	<i>A</i>	<i>B</i>		<i>not-A or not-B</i>
<i>A</i>	<i>B</i>			<i>not-A</i>
<i>Not-A</i>	<i>not-B</i>	✓		<i>not-B</i>
				<i>not-A not-B</i>
				✓

The inference is invalid, as an inspection of the explicit possibilities shows:

<i>If</i>	<i>A</i>	<i>B</i>		<i>not-A or not-B</i>
<i>A</i>	<i>B</i>	x		<i>not-A B</i>
<i>Not-A</i>	<i>not-B</i>	✓		<i>A not-B</i>
<i>Not-A</i>	<i>B</i>	✓		<i>not-A not-B</i>
				✓

For inferences between conditionals and inclusive disjunctions, it happens to be the case that compatible alternatives exist when the premise and conclusion share the polarity of neither term, as Table 1 shows. People should find it difficult to resist these invalid inferences.

The construction of an alternative possibility is not in this case an attempt to discover a counterexample to a putative inference, but an attempt to identify compatible possibilities. The heuristic carries out a consistency check rather than an evaluation of whether the conclusion follows deductively. In the absence of categorical information to allow some alternatives to be eliminated, their number is likely to defeat working memory. There may also be a natural tendency for people to check consistency between assertions (Legrenzi et al., 2003), and it may be amplified when the premise and conclusion share their terms (e.g., A and B in both).

The compatibility heuristic's construction of an alternative possibility is guided by a control process constrained by three assumptions: (a) Only a single alternative is constructed in the search for a compatible alternative. This assumption is based on evidence that people find it difficult to keep in mind multiple alternatives (Johnson-Laird et al., 1992). (b) An alternative possibility is constructed for the conditional only. This assumption is based on evidence that participants find fleshing-out disjunctions difficult, particularly disjunctions that contain negations (e.g., Johnson-Laird et al., 2012). (c) The alternative possibility constructed for the conditional is based on the negation of each of its

mentioned components. This assumption is based on evidence that negating each of the components tends to be the preferred alternative (Jahn et al., 2007), i.e., its contrast class (Oaksford & Stenning, 1992). These assumptions significantly curtail the potential range of compatible possibilities and hence constitute a strong hypothesis leading to specific predictions that can be empirically falsified.

2.2.3. Inferences with no compatible possibilities

Participants should most often reject inferences between conditionals and disjunctions when they cannot think of a compatible initial possibility or compatible alternative, e.g., ‘if Jane got a better job offer she resigned, therefore Jane didn’t get a better job offer or she resigned’. The first steps, envisaging initial possibilities and testing for compatibility, identify no compatible possibilities:

<i>If</i>	<i>A</i>	<i>B</i>	<i>not-A or B</i>
<i>A</i>	<i>B</i>	<i>not-A</i>	
			<i>B</i>
	<i>Not-A</i>	<i>B</i>	

The next steps of constructing the immediately available alternative and again testing for compatibility, still identify no compatible possibilities:

<i>If</i>	<i>A</i>	<i>B</i>	<i>not-A or B</i>
<i>A</i>	<i>B</i>	<i>not-A</i>	
<i>Not-A</i>	<i>not-B</i>		<i>B</i>
	<i>Not-A</i>	<i>B</i>	

Participants’ immediate intuition will be to reject the inference, because they cannot think of a compatible possibility. But the inference is valid:

<i>If</i>	<i>A</i>	<i>B</i>		<i>not-A or B</i>	
<i>A</i>	<i>B</i>	✓	<i>not-A</i>	<i>not-B</i>	✓
<i>Not-A</i>	<i>not-B</i>	✓	<i>A</i>	<i>B</i>	✓
<i>Not-A</i>	<i>B</i>	✓	<i>Not-A</i>	<i>B</i>	✓

To make the inference would require individuals to imagine a further possibility for the conditional or to think through the incomplete possibilities for the disjunction. Of course, when the heuristic fails to deliver a compatible possibility, people may rely on other heuristics, or they may engage in a deliberative process. Hence, even though their immediate intuition may be to reject the inference, some participants may be able to go on to accept it. For valid inferences between conditionals and inclusive disjunctions, the polarity of the second term only is shared by the premise and the conclusion, as Table 1 shows. If people rely on a compatibility heuristic, they should find it difficult to make these valid inferences.

Finally, in some cases rejecting an inference is the correct thing to do, e.g., ‘if the plants were watered, they bloomed, therefore the plants were watered or they didn’t bloom’. The first steps of constructing initial possibilities and testing for compatibility yield no compatible possibilities and nor do the subsequent steps of constructing an alternative:

<i>If</i>	<i>A</i>	<i>B</i>	<i>A or not-B</i>
<i>A</i>	<i>B</i>	<i>A</i>	
<i>Not-A</i>	<i>not-B</i>		<i>not-B</i>
	<i>A</i>	<i>not-B</i>	

The inference is not valid:

<i>If A B</i>			<i>A or not-B or both</i>		
A	B	✓	A	B	✓
Not-A	not-B	✓	not-A	not-B	✓
Not-A	B	x	A	not-B	x

For these inferences, the premise and conclusion share the polarity of the first term only, as [Table 1](#) shows.

Our aim in the 7 experiments we report is to test the new proposal that people rely on a compatibility heuristic to accept or reject inferences between conditionals and disjunctions. If they do, we predict a robust compatibility effect: participants should endorse inferences for which they can construct a compatible initial possibility or a compatible alternative; they should reject inferences for which they cannot construct compatible possibilities. We also consider the predictions made by an alternative theory of reasoning.

2.3. Probabilities

Consider the claim, ‘the Costa Concordia captain intentionally abandoned ship before his passengers, or he is innocent of the crimes for which he is accused’. Is it valid to infer, ‘if the captain didn’t intentionally abandon ship, he is innocent’? The validity of the inference, A or B, therefore if not-A B, is contended (e.g., [Johnson-Laird & Byrne, 2002](#); [Oberauer et al., 2011](#); [Ormerod & Richardson, 2003](#); [Over, Evans, & Elqayam, 2010](#)). The contention reflects the absence of consensus about the cognitive processes that underlie even simple inferences (e.g., [Byrne & Johnson-Laird, 2009](#); [Oberauer, 2006](#)). On the view that people envisage possibilities, the inference is valid, whenever ‘A or B’ is interpreted as an inclusive disjunction and ‘if not-A, B’ is interpreted as a conditional, because the two assertions share all three possibilities:

<i>A or B or both</i>		<i>If not-A B</i>	
A	not-B	not-A	B
Not-A	B	A	not-B
A	B	A	B

The inference between ‘A or B’ and ‘if not-A B’ is valid in either direction, i.e., ‘A or B therefore if not-A, B’ and ‘if not-A, B, therefore A or B’. Accordingly, there are four valid inferences from a conditional to an inclusive disjunction (if A B, not-A or B; if A not-B, not-A or not-B; if not-A B, A or B; if not-A not-B, A or not-B) and the corresponding four inferences from the inclusive disjunction to the conditional are also valid (see [Table 1](#)). The remaining 24 inferences are invalid. The inference is also valid when ‘A or B’ is interpreted as an exclusive disjunction and ‘if not-A B’ is interpreted as a bi-conditional, because the two assertions share their two possibilities:

<i>A or B but not both</i>		<i>If and only if not-A B</i>	
A	not-B	not-A	B
Not-A	B	A	not-B

and in fact, there are twice as many valid inferences between bi-conditionals and exclusive disjunctions as between conditionals and inclusive disjunctions, to which we will return. However, the inference is not valid whenever ‘or’ is interpreted as an exclusive disjunction while ‘if’ is interpreted as a conditional, or whenever ‘or’ is interpreted as inclusive disjunction while ‘if’ is interpreted as any sort of relation other than conditional, such as a bi-conditional or enabling relation; in these cases, the possibilities are not all shared.

An alternative account takes a different view. Logicians have suggested the acceptability of a conditional can be provided by its corresponding conditional probability (e.g., [Adams, 1975](#)). Formulae for ‘fixing... degrees of belief in q given p’ ([Ramsey’s 1929/1990](#)) are provided by Bayesian conditionalization and a probabilistic semantics (e.g., [Adams, 1975](#)). Uncertainty is defined as 1 minus the probability of a

statement, and a valid inference is one in which the uncertainty of its conclusion does not exceed the sum of the uncertainties of its premises. Psychologists have suggested that people understand a conditional 'if A B' by adding the 'if' part, A, to their beliefs and calculating the probability of the 'then' part, B (Evans, 2007; Evans & Over, 2004; Oaksford & Chater, 2007). They think about true antecedents only (A) and not about their belief in the negated 'if' clause (not-A) or whether or not the 'then' clause follows in such circumstances (e.g., Handley, Evans, & Thompson, 2006; Over, Hadjichristidis, Evans, Handley, & Sloman, 2007).

On this account, it is invalid to make an inference from the disjunction 'A or B' to the conditional 'If not-A, B', based on the equation for the general relation between the probability of 'A or B', $P(A \text{ or } B)$ and 'if not-A B', i.e., B given not-A, $P(B|\text{not-A})$:

$$P(A \text{ or } B) = P(A) + P(B|\text{not-A}) - P(A) \times P(B|\text{not-A})$$

because the probability of 'A or B' can be higher than the probability of 'if not-A B' (e.g., Oberauer et al., 2011; Over et al., 2010). However, it is valid to make the inference in the opposite direction 'If not-A B, Therefore A or B'. The probability of the conditional, given by $P(B|\text{not-A})$, can be expanded:

$$P(A) \times P(B|\text{not-A}) + P(\text{not-A}) \times P(B|\text{not-A})$$

and the probability of the disjunction can be given as:

$$P(A) + P(\text{not-A}) \times P(B|\text{not-A})$$

The validity of the inference can be demonstrated by the observation, 'The second addend in this equation is the same as in the expanded equation for the conditional, and the first addend, $P(A)$, can never be smaller than the first addend in the equation for the conditional, $P(A) \times P(B|\text{not-A})$ ' (Oberauer et al., 2011, p. 97). Hence there is an asymmetry in the validity of the two inferences, 'With coherent probability judgments, p (if not- p then q) must be less than or equal to $P(p \text{ or } q)$, by the conditional probability hypothesis. This result in turn means that, although it is logically invalid to infer 'if not- p then q ' (understood as the suppositional conditional) from ' $p \text{ or } q$ ', it is logically valid to infer ' $p \text{ or } q$ ' from 'if not- p then q .' (Over, Evans, & Elqayam, 2010, p. 143). One consequence is that people should make the (valid) inference from the conditional to the disjunction more often than they make the (invalid) inference from the disjunction to the conditional (Oberauer et al., 2011), although the latter inference may be probabilistically strong given a person's prior beliefs (Over et al., 2010). Our interest is in participants' tendency to endorse certain inferences, whether the inferences are considered valid or invalid on different accounts or not. However, we also report the frequency of valid and invalid inferences, as defined within propositional logic.

3. Experiment 1a and 1b: Conditionals and inclusive disjunctions

The aim of the experiments was to test three novel predictions of the compatibility heuristic:

- Participants should accept more inferences when the initial possibilities they think about for a conditional and disjunction are compatible, i.e., when the premise and conclusion share the polarity of both terms, e.g., if A then B, therefore A or B or both.
- They should accept more inferences when the most readily available alternative possibilities they think about are compatible, i.e., when the premise and conclusion share the polarity of neither term, e.g., if A then B, therefore not-A or not-B or both.
- They should accept more inferences from problems for which they can envisage compatible initial possibilities, than problems for which they can envisage compatible alternatives, given that the latter require an additional step.

The participants' task was to judge whether the conclusion followed from the premises. In Experiment 1a we provided them with two response options, yes and no, and in Experiment 1b with three responses options, yes, no, and it is not possible to know. The number of response options has been argued to affect the inferences people make in some circumstances (e.g., Evans & Over, 2004). In this

case the two experiments revealed the same results and so we report them together. We examined the complete set of 32 inferences between conditionals of any one of the four possible polarities and disjunctions of any one of the four possible polarities (see Table 1), to control for any idiosyncrasies in individual forms, such as ‘polarity biases’ created by negation (e.g., Evans & Over, 2004).

3.1. Method

3.1.1. Participants and procedure

The participants who took part in the experiment were undergraduate students at the University of La Laguna, Tenerife, Spain. In Experiment 1a 64 participants volunteered to take part, and a different set of 60 participants volunteered in Experiment 1b. None of the participants in any of the experiments reported having received formal training in logic. Participants were tested in large groups and received the instructions and problems in a booklet that they completed at their own pace.

3.1.2. Design and materials

Participants acted as their own controls and completed 32 problems (see Table 1). The problems consisted of conditionals in all four possible polarities (A B, A not-B, not-A B, not-A not-B) and inclusive disjunctions in all four polarities, resulting in 16 distinct forms, which were presented either with a conditional premise and disjunctive conclusion, or a disjunctive premise and conditional conclusion. The assertions had explicitly negated terms, e.g., ‘if there is not a lion in the zoo then there is a tiger. Therefore there is a lion in the zoo or there is not a tiger or both’. Each of the 32 problems was instantiated in a different content drawn from four different domains, animals, fruit, e.g., ‘if there is a banana in the fruit bowl then there is a melon’, flowers, e.g., ‘if there is a dahlia in the garden then there is a rose’ and kitchen implements, e.g., ‘if there is a knife on the table then there is a plate’. The contents were designed to be ‘non-constructive’, i.e., not based on participants’ prior beliefs. The design had the within-participants variables of problem type, with 2 levels: conditional to disjunction inference vs disjunction to conditional inference, and compatibility, identified by the shared polarity of terms in the premise and conclusion, with 4 levels: shared both terms (compatible initial possibility), neither (compatible alternative), first term only (no compatible initial or alternative possibility), second term only (no compatible initial or alternative possibility). Half of the participants completed the 16 problems based on a disjunctive premise with a conditional conclusion first, and then they completed the 16 problems based on a conditional premise with a disjunctive conclusion; the other half completed the problems in the opposite order. The problems were presented in a different randomized order within each block for each participant.

The measure was whether or not participants accepted the presented conclusion as valid, or rejected it. In Experiment 1a they were given the following instructions illustrated by an example not used in the experiment: “For each problem you must indicate whether or not the conclusion necessarily follows given the premise that precedes it. If you think that the conclusion follows from the premise you should mark the option ‘yes’, if you think that the conclusion does not follow from the premise you should mark the option ‘no’. A conclusion is necessarily true when the conclusion must follow given the truth of the premise.” In Experiment 1a, participants were given two response options: yes, no; in Experiment 1b, they were given three response options: yes, no, it is not possible to know; they were asked to indicate their response by circling one option for each problem.

3.2. Results and discussion

A mixed-design ANOVA for each experiment, with a design of 2 (order of presentation: IF therefore OR problem set first vs OR therefore IF problem set first) \times 2 (problem type: IF therefore OR vs OR therefore IF), by 4 (compatibility: shared premise and conclusion polarity for both terms: compatible initial possibility, neither term: compatible alternative, first term only, second term only), with repeated measures on the second two factors used the Greenhouse–Geisser correction for the violation of sphericity assumption. The order in which participants completed the problems had no effect (in these or any other of the experiments) and did not interact with any variable in the statistical analyses.

The ANOVAs showed a main effect of compatibility, in Experiment 1a, $F(2.259, 140.06) = 112.38$, $Mse = .12$, $p < .001$, $\eta_p^2 = .64$, and Experiment 1b, $F(2.204, 127.854) = 27.49$, $Mse = .16$, $p < .001$, $\eta_p^2 = .32$, due to the following differences:

- (a) Participants endorsed more inferences that had compatible initial possibilities – the premise and conclusion shared both terms, compared to a first term only, in Experiment 1a, 84% vs 21% $F(1, 63) = 273.55$, $Mse = .05$, $p < .001$, $\eta_p^2 = .81$, and in Experiment 1b, 62% vs 26%, $F(1, 59) = 48.23$, $Mse = .08$, $p < .001$, $\eta_p^2 = .45$, and compared to a second term only, in Experiment 1a, 84% vs 30%, $F(1, 63) = 155.69$, $Mse = .06$, $p < .001$, $\eta_p^2 = .71$, and in Experiment 1b, 62% vs 30%, $F(1, 59) = 46.20$, $Mse = .07$, $p < .001$, $\eta_p^2 = .44$.
- (b) They endorsed more inferences that had compatible alternatives – the premise and conclusion shared neither term, compared to a first term only, in Experiment 1a, 55% vs 21%, $F(1, 63) = 71.69$, $Mse = .05$, $p < .001$, $\eta_p^2 = .53$, and in Experiment 1b, 46% vs 26%, $F(1, 59) = 18.66$, $Mse = .07$, $p < .001$, $\eta_p^2 = .24$, or a second term only, in Experiment 1a, 55% vs 29% $F(1, 63) = 37.30$, $Mse = .06$, $p < .001$, $\eta_p^2 = .37$, and in Experiment 1b, 46% vs 30%, $F(1, 59) = 11.96$, $Mse = .07$, $p < .002$, $\eta_p^2 = .17$.
- (c) They made more inferences that had compatible initial possibilities (both terms) compared to compatible alternatives (neither terms), in Experiment 1a, 84% vs 55%, $F(1, 63) = 61.02$, $Mse = .04$, $p < 0.001$, $\eta_p^2 = .49$, and in Experiment 1b, 62% vs 46%, $F(1, 59) = 14.74$, $Mse = .05$, $p < .001$, $\eta_p^2 = .20$.

There was no main effect of problem type, in Experiment 1a, $F(1, 62) = 1.42$, $Mse = .05$, $p = .24$, $\eta_p^2 = .02$, and Experiment 1b, $F(1, 58) = 1.34$, $Mse = .06$, $p = .25$, $\eta_p^2 = .02$, but compatibility and problem type interacted in Experiment 1a, $F(2.350, 145.712) = 3.39$, $Mse = .06$, $p < .030$, $\eta_p^2 = .05$ and marginally so in Experiment 1b, $F(2.373, 137.613) = 2.43$, $Mse = .07$, $p = .08$, $\eta_p^2 = .04$. The interaction in Experiment 1a shows that the compatibility effect occurs equally for inferences from conditionals to disjunctions, and for inferences from disjunctions to conditionals, and the only difference is that more inferences are made from conditionals to disjunctions than from disjunctions to conditionals, for inferences that had compatible alternatives (neither term), 61% vs 49%, $F(1, 63) = 6.86$, $Mse = .07$, $p < .02$, $\eta_p^2 = .10$. There were no differences for the other three cases, i.e., both terms, first term only, or second term only.

A similar trend was observed for each one of the 32 forms of inference in each experiment, as Fig. 1a and b show. This trend occurred in almost every one of the 64 observations plotted for the two experiments. The main exception is a very notable one: for the affirmative disjunction, ‘A or B or both’, the valid inference ‘therefore if not A then B’ was made as often as the inference for which there is a compatible initial possibility, ‘therefore if A then B’ in Experiment 1b (48% vs 45%), and as often as the inference for which there is a compatible alternative, ‘therefore if not-A then not-B’ in Experiment 1a (44% vs 41%) and Experiment 1b (48% vs 32%). The inference, which is valid according to possibility accounts (Johnson-Laird & Byrne, 2002) but invalid according to probability accounts (Over et al., 2010), is endorsed frequently. It is also notable that participants made more of it compared to ‘if not-A then B, therefore A or B’ (44% vs 30% in Experiment 1a and 48% vs 28% in Experiment 1b). The result is consistent with earlier findings (Oberauer et al., 2011, p. 104). According to the probability account, the latter inference is valid and the former invalid, and so this trend is the opposite of its expectations (Over et al., 2010; see Oberauer et al., 2011). The materials in Experiment 1a and 1b were chosen to be neutral and pragmatic factors cannot explain the asymmetry (Oberauer et al., 2011, p. 114).

The only other exceptions to the overall trend in Fig. 1a and b were two cases in Experiment 1b, where the inference for which there was a compatible alternative (neither terms) was made only as often as, not more often than, the valid inference, for the premise ‘if not-A, not-B’ and the premise ‘not-A or not-B’. The discrepancy may reflect a difficulty with negations (see also Oberauer et al., 2011). We note that participants made fewer valid inferences between a negated disjunction ‘not-A or B’ and a conditional ‘if A, B’, compared to inferences between an affirmative disjunction, ‘A or B’ and a conditional ‘if not-A, B’ (16% vs 37% in Experiment 1a and 16% vs 38% in Experiment 1b),

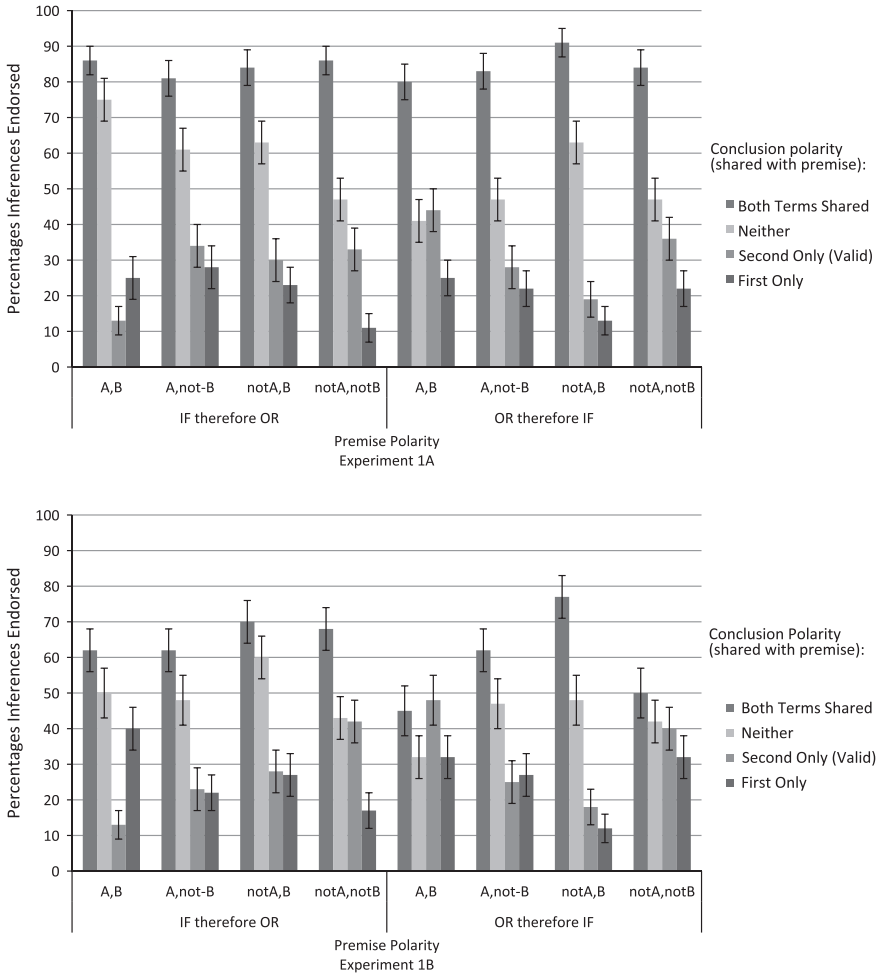


Fig. 1. (a and b) The percentages of inferences endorsed between conditionals and inclusive disjunctions when the premise and conclusion share the polarity of both terms (compatible initial possibility), neither (compatible alternative), the first only, or the second only (this latter is the valid case), in Experiment 1a for two response options and Experiment 1b for three response options. Error bars are standard error of the mean.

consistent with the difference observed by Oberauer et al. (2011). However, our results show further that the difficulty is not a general one with negated disjunctions, instead it is confined to disjunctions with a negated first disjunct. Participants did not make fewer valid inferences between the disjunction with a negated second disjunct, ‘A or not-B’ and the conditional ‘if not-A, not-B’ (31% in Experiment 1a and 34% in Experiment 1b) nor between the disjunction with two negated disjuncts, ‘not-A or not-B’ and the conditional ‘if A, not-B’ (35% in Experiment 1a and 32% in Experiment 1b).

The few previous studies of inferences between conditionals and disjunctions examined only two or three instances of the 32 possible forms of inference, and provided conflicting results: people made more inferences from conditionals to disjunctions than inferences from disjunctions to conditionals (Ormerod & Richardson, 2003), or the opposite (Oberauer et al., 2011). Our results show that for all 32 forms of inference overall, there is no difference in the frequency of inferences accepted from conditionals to disjunctions vs from disjunctions to conditionals, 49% vs 46% in Experiment 1a and 42%

vs 40% Experiment 1b, as the absence of a main effect of problem type indicates. However, we carried out a further comparison that showed the validity of the inference is relevant (with validity defined according to the propositional calculus). Participants made more logically correct inferences from a disjunction to a conditional than from a conditional to a disjunction, in Experiment 1a (yes to valid problems and no to invalid ones) 44% vs 40%, $F(1,63) = 5.83$, $Mse = .01$, $p < .02$, $\eta_p^2 = .09$, and in Experiment 1b (yes to valid problems and no, or it is not possible to know, to invalid ones), 52% vs 46%, $F(1,59) = 6.68$, $Mse = .01$, $p < .02$, $\eta_p^2 = .10$.

The results provide little support for the view that a conditional is interpreted as a material implication, i.e., the conditional is true whenever its antecedent is false or its consequent is true, a view captured in formal inference rule theories (e.g., Braine & O'Brien, 1998; Chevallier et al., 2008; Rips, 1994). On a truth functional account, the material implication conditional 'if A B' is logically equivalent to an inclusive disjunction, 'not A or B' (e.g., Jeffrey, 1981). Hence, the inference 'A or B, therefore if not-A, B' is valid because the premise and conclusion are equivalent. One reason why inferences between conditionals and disjunctions have been considered important is that they test whether the interpretation of the conditional is the material implication. Given that participants did not endorse the logically valid inferences more often than the invalid ones, the data rule out this idea. For the same reason, the data provide little support for the idea that people construct fully explicit mental models to make these inferences.

The results corroborated the predictions of the compatibility heuristic. Participants frequently endorsed inferences when the initial possibilities were compatible – the premise and conclusion shared both terms (84% in Experiment 1a, 62% in Experiment 1b), or when the alternative possibilities were compatible – the premise and conclusion shared neither term (55% in Experiment 1a, 46% in Experiment 1b). They rejected inferences when they could not envisage a compatible possibility – i.e., when the premise and conclusion shared only the first term (21% in Experiment 1a, 26% in Experiment 1b), or only the second term (30% in Experiment 1a, 30% in Experiment 1b), and the acceptance of inferences in these latter cases may reflect the operation of a deliberative process, or of other heuristic processes.

The compatibility heuristic engages a third step when it does not identify a compatible possibility, to construct an immediately available alternative, based on the opposite of the elements mentioned in the conditional. This assumption curtails the range of compatible possibilities and the results provide strong support for it. An alternative proposal is, 'people don't construct separate sets of mental models for the two statements and compare them with each other' (Oberauer et al., 2011, p. 116), because models are not constructed for the conditional: 'its meaning cannot be captured by a set of models, but ... it provides constraints for models that represent other statements' (Oberauer et al., 2011, p. 116). Hence, 'the construction of mental models must start from the disjunctive statement' (Oberauer et al., 2011, p. 116) and, 'people could construct a set of mental models representing the meaning of the disjunction, and check whether it is compatible with the conditional' (Oberauer et al., 2011, p. 116). An example is, 'when asked whether "A or C" follows from "If not A then C", or the other way round, people could construct the two mental models [not-A C, A not-C] to represent the meaning of the disjunction, and check whether the conditional is consistent with it.' (Oberauer et al., 2011, p. 116). To check whether the conditional is consistent with it requires that 'every model of the antecedent should be accompanied by a model of the consequent; this is the case in the two models above. Based on this reasoning, people could conclude that the direct inference from one to the other is valid.' (Oberauer et al., 2011, p. 116). This disjunction-led procedure makes the opposite predictions to the compatibility heuristic for each of the 32 inferences. For example, it predicts for the first inference in Table 1: 'A or B or both, if A, B' that people represent several complete possibilities for the disjunction (A not-B, not-A B) and they check that every model of the conditional's antecedent is accompanied by a model of its consequent (A B), and since that is *not* the case, they should reject the inference. But as Fig. 1 shows 80% of participants made it. It predicts for the second inference in Table 1: 'A or B or both, if A, not-B' that people represent the disjunction (A not-B, not-A B) and check every model of the conditional's antecedent is accompanied by one of its consequent (A not-B), and since that is the case, they should accept the inference. But as Fig. 1 shows only 25% of participants do so. Its predictions are the mirror-image of the compatibility heuristic's, and go against the data for each of the 32 inferences, as Fig. 1 shows. A potential modification is that people represent fully

Table 2

The consistent possibilities for 16 forms of inference between an ‘A only if B’ conditional and inclusive disjunction used in Experiments 2.

	Consistent Possibilities	A only if B	A only if not-B	not-A only if B	not-A only if not-B
Consistent Possibilities		B & A Not-B & not-A B & not-A	Not-B & A B & not-A Not-B & not-A	B & not-A Not-B & A B & A	Not-B & not-A B & A Not-B & A
A or B or both	A & not-B <i>Not-A & B</i> A & B	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: First Term Only	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Terms Compatible Initial Possibility
A or not-B or both	A & B <i>Not-A & not-B</i> A & not-B	Shared Polarity: First Term Only	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: Neither Terms Compatible Initial Possibility	Shared Polarity: Second Term Only Valid
Not-A or B or both	Not-A & not-B A & B Not-A & B	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Terms Compatible Initial Possibility	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: First Term Only
Not-A or not-B or both	Not-A & B A & not-B Not-A & not-B	Shared Polarity: Neither Terms Compatible Initial Possibility	Shared Polarity: Second Term Only Valid	Shared Polarity: First Term Only	Shared Polarity: Both Terms Compatible Initial Possibility

Key: The initial possibilities considered for an ‘A only if B’ conditional and a disjunction are indicated in bold font, and the first alternative in italic font. The 8 inferences with a compatible initial possibility are indicated. The shared polarity of the terms in the premise and conclusion is provided: both terms (compatible initial possibility), neither term (compatible alternative), first term only, or second term only. The 4 valid inferences are also indicated.

complete possibilities for an explicit ‘A or B or both’ inclusive disjunction (A not-B, not-A B, A B), even though people reason ‘without noticing that “A or C” is compatible with the A and C conjunction’ (Oberauer et al., 2011, p. 116), because ‘the full set of models for the inclusive disjunction is never constructed in this task’ (Oberauer et al., 2011, p. 116). However, this modified disjunction-led procedure predicts that participants should endorse only 8 of the 32 inferences (those that correspond to the logically valid inferences). For example, it predicts for the second inference in Table 1, ‘A or B or both, if A, not-B’ that people represent the disjunction (A not-B, not-A B, A B) and check every model of the conditional’s antecedent is accompanied by one of its consequent (A not-B), and since that is now *not* the case, they should reject the inference. But it predicts for the third inference in Table 1, ‘A or B or both, if not-A then B’ that people represent the disjunction (A not-B, not-A B, A B) and check every model of the conditional’s antecedent is accompanied by one of its consequent (not-A B), and since that is the case, they should accept the inference. But participants do not tend to accept these 8 inferences and reject the remaining 24 inferences: its predictions once again go against the data, as Fig. 1 shows.

The compatibility effect was observed when participants had the option to accept or reject the inference in Experiment 1a, and also when they had the option to judge it was not possible to know, in Experiment 1b. Importantly, the results distinguish the compatibility heuristic from a superficial matching of the linguistic polarity of terms, as participants did not merely make more inferences when both polarities matched, but also when neither did. In the next experiment, we show the

compatibility effect is accentuated when characteristics of the inference increase the compatible possibilities.

4. Experiment 2: ‘Only if’ conditionals and disjunctions

The aim of the experiment was to test a novel prediction of the compatibility heuristic for inferences between ‘A only if B’ conditionals and disjunctions, such as ‘I’ll go to the party only if I get my essay done, so I won’t go to the party or I won’t get my essay done’. Participants should tend to accept as many inferences from problems in which the premise and conclusion share neither term, as for problems in which they share both terms. The prediction is a consequence of the observation that people envisage two initial possibilities for ‘A only if B’ from the outset.

Logicians have identified that ‘there is an apple only if there is a melon’ (A only if B) is logically equivalent to ‘if there is an apple, there is a melon’ (if A B). The equivalence can be illustrated by the observation that the one situation that renders either conditional false is the situation in which there is an apple and no melon (A and not-B) (Jeffrey, 1981). But people interpret conditionals based on ‘if’ and ‘only if’ differently (e.g., Cheng & Holyoak, 1985; Evans, 1977; Johnson-Laird & Byrne, 1989). Evidence from the inferences participants make, and their reading times, suggests they envisage two possibilities from the outset for A only if B, i.e., ‘there is an apple and there is a melon’ (A and B) and ‘there is not an apple and there is not a melon’ (not-A and not-B) (Johnson-Laird & Byrne, 1989; Santamaria & Espino, 2002). The evidence also indicates they envisage the elements in the opposite direction to their order of mention, i.e., ‘there is a melon and there is an apple’ (B to A) (Egan, Garcia-Madruga, & Byrne, 2009; Evans, 1993):

<i>A only if B</i>	
B	A
Not-B	not-A

An ‘only if’ assertion seems to work best when its second component (B) precedes in time its first component (A), and it often contains a precondition in its consequent, e.g. ‘you can go out to play only if you tidy your room’ (Cheng & Holyoak, 1985; Girotto, Mazzocco, & Cherubini, 1992). Accordingly, the compatibility heuristic immediately detects compatible initial possibilities for inferences between ‘A only if B’ conditionals and disjunctions not only when they share both terms, but also when they share neither term, as Table 2 shows.

The first step of the compatibility heuristic for the inference about the party and the essay is that two possibilities are envisaged for the conditional from the outset:

<i>A only if B</i>		<i>not-A or not-B</i>	
B	A	not-A	
Not-B	not-A	✓	not-B
		Not-A	not-B
			✓

The second step is that the compatibility heuristic can identify a compatible possibility in these initial possibilities: not-A and not-B. Because two possibilities are envisaged for ‘A only if B’ from the outset, the premise and the conclusion have compatible initial possibilities not only when they share both terms but also when they share neither term, as Table 2 shows. Step 3, that of considering an alternative, is not required for ‘A only if B’ problems when they share neither term, unlike for ‘if A B’ problems.

Of course, since the terms are represented in the opposite direction for ‘A only if B’ conditionals (B to A) than for disjunctions (A to B), participants may detect somewhat fewer compatible possibilities overall. Nonetheless, the heuristic leads to the novel prediction that participants should accept as many inferences when the premise and conclusion shares neither term as when they share both terms. The participants’ task was once again to judge the validity of the conclusion, and we provided them with two response options (yes and no) since the number of response options made no difference in the previous experiments.

4.1. Method

4.1.1. Participants, design, procedure, and materials

The participants were a new set of 32 undergraduates at the University of La Laguna. The design and procedure were the same as the previous experiments, and the materials were the same with the exception that the conditionals were based on ‘A only if B’.

4.2. Results and discussion

An ANOVA similar to those in the previous experiments showed a main effect of compatibility, $F(2,306,69.179) = 31.59$, $Mse = .09$, $p < .001$, $\eta_p^2 = .51$, due to the following differences:

- (a) Participants endorsed more inferences that had compatible initial possibilities, i.e., the premise and conclusion shared both terms, compared to a first term only, 62% vs 27% $F(1,31) = 51.33$, $Mse = .04$, $p < .001$, $\eta_p^2 = .62$; or a second term only, 62% vs 29%, $F(1,31) = 46.97$, $Mse = .04$, $p < .001$, $\eta_p^2 = .60$. These results replicate the previous experiments.
- (b) They endorsed more inferences that had compatible initial possibilities, this time when the premise and conclusion shared neither term, compared to a first term only, 56% vs 27%, $F(1,31) = 32.97$, $Mse = .04$, $p < .001$, $\eta_p^2 = .52$, or a second term only, 56% vs 29%, $F(1,31) = 25.88$, $Mse = .05$, $p < .001$, $\eta_p^2 = .46$. These results also replicate the previous experiments.
- (c) There were no differences between inferences that had compatible initial possibilities as a result of the premise and conclusion sharing both terms vs those that shared neither term, 62% vs 56%, $F(1,31) = 2.47$, $Mse = .02$, $p = .13$, $\eta_p^2 = .07$. This result supports the novel prediction that participants endorse as many inferences when the problems share both terms or neither term, since two possibilities are envisaged from the outset for ‘A only if B’.

There was a main effect of problem type, $F(1,30) = 14.00$, $Mse = .06$, $p < .002$, $\eta_p^2 = .32$, and problem type did not interact with compatibility, $F < 1$. Participants made more inferences overall from an ‘A only if B’ conditional to a disjunction than they did from a disjunction to the conditional, 49% vs 38%, as the main effect of problem type shows. They made as many *logically correct* inferences from an ‘A only if B’ conditional to a disjunction as they did from a disjunction to the conditional, 43% vs 49%, $F(1,31) = 2.91$, $Mse = .02$, $p = .10$, $\eta_p^2 = .09$.

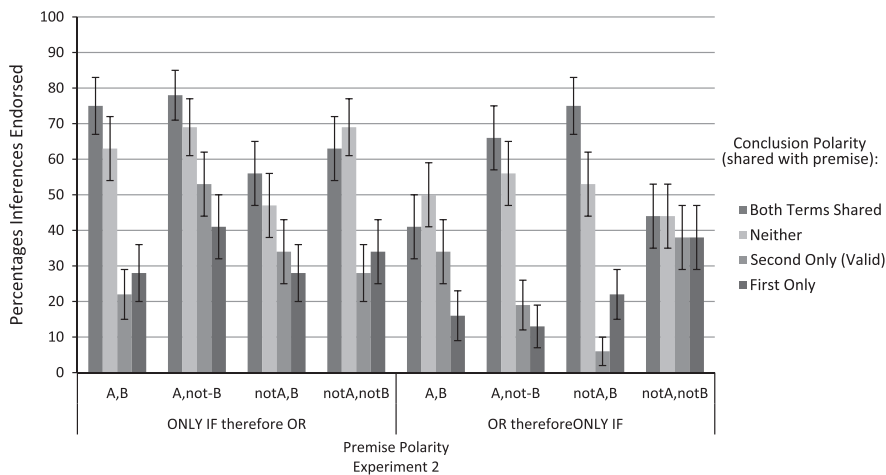


Fig. 2. The percentages of inferences endorsed between ‘A only if B’ conditionals and inclusive disjunctions when the premise and conclusion share the polarity of both terms or neither term (compatible initial possibility in both cases), the first only, or the second only in Experiment 2. Error bars are standard error of the mean.

A similar trend was observed for each of the individual inferences, in almost every one of the 32 observations plotted in Fig. 2. It is again notable that the exception to this trend is for the affirmative disjunction, 'A or B or both'. The valid inference to 'therefore not-A only if B' was made almost as often as the inference for which there is a compatible initial possibility based on sharing both terms, 'therefore A only if B' (34% vs 41%). Once again the inference was made often. It is again notable that participants made the inference from the disjunction to the conditional, 'A or B therefore not-A only if B' as readily as from the conditional to the disjunction, 'not-A only if B, therefore A or B' (34% in each case). According to the probability account, the latter inference is valid and the former invalid, and so this trend is the opposite of its expectations (Over et al., 2010; see Oberauer et al., 2011).

The results replicate those of the previous experiments, and extend them to the logical form of 'A only if B' to show that the compatibility heuristic can be amplified by characteristics of the problems. Participants frequently endorsed inferences between 'A only if B' conditionals and disjunctions when they imagined a compatible initial possibility – in this case when the premise and conclusion shared both terms, 62%, or neither term, 56%. Fewer inferences were endorsed when they could not imagine a compatible possibility – when the premise and conclusion shared only the first term, 27% or only the second term, 29%, and these endorsements may reflect the operation of a deliberative process or of other heuristic processes. The results distinguish the compatibility heuristic from a superficial matching of the linguistic polarity of terms, as participants did not merely make more inferences when both polarities matched, but also when neither did. The next experiment shows that the compatibility effect is eliminated when the compatibility heuristic identifies no compatible possibilities.

5. Experiment 3: Bi-conditionals and exclusive disjunctions

We test the novel prediction that there should be no compatibility effect for inferences between bi-conditionals and exclusive disjunctions, e.g., 'if and only if he went for a walk, he had a coffee, therefore he went for a walk or he had a coffee but not both' (if and only if A, B, therefore A or B but not both). The first steps of thinking of the initial possibilities and carrying out a compatibility test do not identify a compatible possibility:

<i>If and only if A, B</i>	<i>A or B but not both</i>
A B	A
	B

The subsequent steps of thinking of the immediately available alternative and carrying out a compatibility test also do not identify a compatible possibility:

<i>If and only if A, B</i>	<i>A or B but not both</i>
A B A	
Not-A not B B	

In fact, the compatibility heuristic does not identify a compatible possibility or compatible alternative for any one of the 32 inferences between bi-conditionals and exclusive disjunctions, as Table 3 shows.

Consider another inference, 'if and only if you don't buy a parking ticket, you're clamped. Therefore, you buy a parking ticket or you're clamped but not both' (if and only if not A, B, therefore A or B but not both). The compatibility heuristic does not identify a compatible initial possibility or a compatible alternative:

<i>If and only if not-A B</i>	<i>A or B but not both</i>
Not-A B	A
A not B	B

But the inference is valid, as a deliberative process that considers the fully explicit possibilities could identify:

Table 3

The consistent possibilities for 16 forms of inference between a bi-conditional and exclusive disjunction in Experiment 3.

	Consistent Possibilities	If and only if A B	If and only if A not-B	If and only if not-A B	If and only if not-A not-B
Consistent Possibilities		A & B <i>Not-A & not-B</i>	A & not-B <i>Not-A & B</i>	Not-A & B <i>A & not-B</i>	Not-A & not-B <i>A & B</i>
A or B but not both	A & not-B Not-A & B	Shared Polarity: Both Terms	Shared Polarity: First Term Only Valid	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Term
A or not-B but not both	A & B Not-A & not-B	Shared Polarity: First Term Only Valid	Shared Polarity: Both Terms	Shared Polarity: Neither Term	Shared Polarity: Second Term Only Valid
Not-A or B but not both	Not-A & not-B A & B	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Term	Shared Polarity: Both Terms	Shared Polarity: First Term Only Valid
Not-A or not-B but not both	Not-A & B A & not-B	Shared Polarity: Neither Term	Shared Polarity: Second Term Only Valid	Shared Polarity: First Term Only Valid	Shared Polarity: Both Terms

Key: The initial possibilities considered for a bi-conditional and an exclusive disjunction are indicated in bold font, and the first alternative in italic font. The shared polarity of the terms in the premise and conclusion is provided: both terms, neither term, first term only, or second term only. The 8 valid inferences are also indicated.

<i>If and only if not-A B</i>		<i>A or B but not both</i>	
Not-A B	✓	A not-B	✓
A not-B	✓	not-A B	✓

Given that the premise and the conclusion are each consistent with only two possibilities, rather than the three possibilities each for an inference between a conditional and inclusive disjunction, participants may be more likely to think through the possibilities and engage in a deliberative process.

Accordingly, we test the following two new predictions:

- (a) There should be no compatibility effect for inferences between bi-conditionals and exclusive disjunctions, i.e., no systematic pattern in the inferences made when the premise and conclusion share both terms, neither term, or only the first or second term, given that the compatibility heuristic does not identify compatible initial possibilities or alternatives.
- (b) Inferences between bi-conditionals and exclusive disjunctions should be made accurately given that they are based on two possibilities each for the premise and conclusion, rather than three possibilities.

5.1. Method

5.1.1. Participants, procedure, design and materials

The new set of 32 participants who took part in the experiment were undergraduates at the University of La Laguna. The design and procedure were the same as the previous experiments. The mate-

rials were the same with the exception that bi-conditionals ‘if and only if A then B’ were used instead of conditionals, and exclusive disjunctions ‘either A or B but not both’ were used instead of inclusive disjunctions.

5.2. Results and discussion

An ANOVA similar in design to those in the previous experiments showed that there was no main effect of compatibility, $F(1.526, 45.790) = 2.39$, $Mse = .23$, $p = .12$, $\eta_p^2 = .07$. The result shows no systematic pattern of differences occurs between inferences for which the premise and conclusion share both terms, neither terms, the first term only or the second term only. It corroborates the prediction of the compatibility heuristic, which does not identify a compatible possibility in any of these cases.

There was a main effect for problem type, $F(1, 30) = 14.36$, $Mse = .04$, $p < .002$, $\eta_p^2 = .32$, and problem type and compatibility did not interact, $F < 1$. Participants endorsed more inferences overall from an exclusive disjunction to a bi-conditional, than from a bi-conditional to an exclusive disjunction, 52% vs 43%, as the main effect of problem type showed. They made as many *logically correct* inferences from a bi-conditional to an exclusive disjunction, as they did from an exclusive disjunction to a bi-conditional, 53% vs 57%, $F(1, 31) = .90$, $Mse = .03$, $p = .35$, $\eta_p^2 = .03$.

The second prediction was also corroborated: participants made more logically correct inferences from bi-conditionals to exclusive disjunctions in this experiment compared to the frequency of correct inferences from conditionals to inclusive disjunctions in Experiment 1a, 53% vs 40%, $F(1, 94) = 16.79$, $Mse = .37$, $p < .001$, $\eta_p^2 = .15$, and they made more logically correct inferences from exclusive disjunctions to bi-conditionals in this experiment compared to the inferences from inclusive disjunctions to conditionals in Experiment 1a, 57% vs 44%, $F(1, 94) = 11.43$, $Mse = .33$, $p < .002$, $\eta_p^2 = .11$. As Fig. 3 shows, participants frequently endorsed the valid inferences (for this set of problems, inferences where the premise and conclusion share a first term only, or a second term only), particularly for inferences from exclusive disjunctions to bi-conditionals. The result is consistent with the idea that participants can more readily think through the possibilities for bi-conditionals and exclusive disjunctions (two consistent possibilities each) compared to conditionals and inclusive disjunctions (three possibilities each). The result may be difficult to explain on a formal inference-rule view. Inferences based on bi-conditionals and exclusive disjunctions should require similar inference rules to inferences based on conditionals and inclusive disjunctions, as well as similar sorts of derivations, and similar numbers of steps (e.g., Braine & O’Brien, 1998; Rips, 1994; see Ormerod & Richardson, 2003).

It is noteworthy that participants frequently made the valid inferences from exclusive disjunctions, regardless of the presence of negatives, but they made even more valid inferences from ‘A or B but not both’ as well as ‘not-A or not-B but not both’. The pattern may reflect, in part, a difficulty that participants experience in understanding exclusive disjunctions that contain negation, e.g., ‘A or not-B but not both’ (Elqayam, Ohm, Evans, & Over, 2010; see also Oberauer et al., 2011). In contrast, from the bi-conditionals ‘if and only if A, B’ and ‘if and only if not-A, not-B’ they rarely endorsed any conclusions at all. Overall, it is evident that a very different pattern of responses was observed for inferences between bi-conditionals and exclusive disjunctions, compared to the previous experiments. The pattern is suggestive of an attempt to deploy a deliberative process.

Participants cannot readily spot a compatible possibility when they think about an inference between a bi-conditional and an exclusive disjunction. They may consider further possibilities consistent with the premise and conclusion, and hence make better inferences. However, the next experiment shows that the compatibility heuristic is relied on widely for inferences between conditionals and inclusive disjunctions, even when the polarity of the terms in the conditional and disjunction is conveyed indirectly.

6. Experiment 4: Implicit negation

A strong test of the compatibility heuristic is to examine whether a compatibility effect occurs even for inferences that do not share referents in common. The assertions in the previous experiments used explicit negation, e.g., ‘There’s a circle on the blackboard or there’s a triangle, therefore if there’s not a

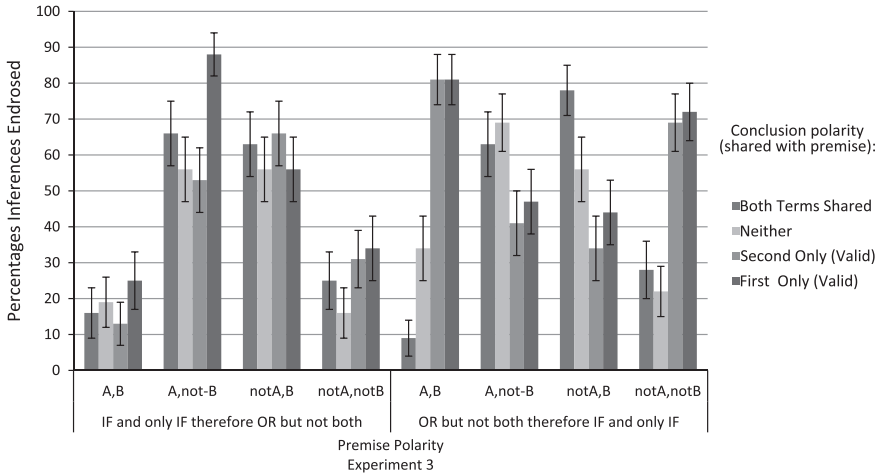


Fig. 3. The percentages of inferences endorsed between bi-conditionals and exclusive disjunctions when in the premise and conclusion share the polarity of both terms, neither, the first only, or the second only, in Experiment 3. Error bars are standard error of the mean.

circle on the blackboard, there's a triangle (A or B, If not-A B). The same inference can be expressed using implicit negation, i.e., 'There's a circle on the blackboard or there's a triangle, therefore if there's a star on the blackboard, there's not a square'. The aim of the experiment was to test the novel prediction that the compatibility heuristic applies even to inferences between conditionals and disjunctions based on implicit negation. The use of indirect co-reference separates the compatibility of possibilities entirely from any superficial matches in the linguistic terms of the premise and conclusion.

Explicit negation (using 'not' before a term) affects the inferences people make compared to implicit negation (using the binary opposite of a term), for conditional inferences. For 'If the letter is not R, the number is 5' (not-A and B), participants can identify the falsifying case: 'the letter is not R and the number is not 5'. But when described using implicit negation, e.g., 'the letter is B and the number is 6', participants are less accurate (e.g., Evans & Over, 2004). They exhibit a 'matching' effect, a tendency to rely on a superficial linguistic match of the terms, regardless of the presence of negation, e.g., 'if the letter is not R, the number is 5' appears to be about R's and 5's (e.g., Evans, 1983, 1998; Evans, Legrenzi, & Girotto, 1999). When the falsifying case is described using implicit negation, B's and 6's, it does not appear to be relevant (e.g., Sperber, Cara, & Girotto, 1995).

In this experiment, an inference such as, 'if there is a tiger in the zoo, there is an eagle. Therefore there is not a tiger or there is not an eagle or both' (if A, B, therefore not-A or not-B or both) was conveyed instead using implicit negation of terms. A binary situation was defined for each content, e.g., 'in the zoo the big cat is a tiger or a lion and the bird is an eagle or a falcon' (A or C, and B or D, where C is not-A and D is not-B) and the assertions then used implicit negation, 'if there is tiger in the zoo, there is an eagle. Therefore, there is a lion or there is a falcon or both.' (if A, B, therefore C or D or both).

Our proposal is that the compatibility heuristic will occur even for implicit negation. The first step of constructing initial possibilities yields no compatible possibilities:

If A B C or D (C = not-A, D = not-B)
 A B C
 D
 C D

Table 4

The consistent possibilities for 16 forms of inference between a conditional and inclusive disjunction using implicit negation of terms (in a binary state of affairs) in Experiment 4.

Binary world: A or C, C=not-A B or D, D=not-B	Consistent Possibilities	If A B	If A not B	If not-A B	If not-A not B
Consistent Possibilities		A & B <i>Not-A & not-B</i> Not-A & B	A & not-B <i>Not-A & B</i> Not-A & not-B	Not-A & B <i>A & not-B</i> A & B	Not-A & not-B <i>A & B</i> A & not-B
Not C or Not D or both (\exists A or B or both)	Not-C & D C & not-D Not-C & not-D	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: First Term Only	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Terms <i>Compatible Alternative</i>
Not C or D or both (\exists A or not B or both)	Not-C & not-D C & D Not-C & D	Shared Polarity: First Term Only	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: Neither Terms <i>Compatible Alternative</i>	Shared Polarity: Second Term Only Valid
C or not D or both (\exists not A or B or both)	C & D Not-C & not-D C & not-D	Shared Polarity: Second Term Only Valid	Shared Polarity: Neither Terms <i>Compatible Alternative</i>	Shared Polarity: Both Terms Compatible Initial Possibility	Shared Polarity: First Term Only
C or D or both (\exists not A or not B or both)	C & not-D Not-C & D C & D	Shared Polarity: Neither Terms <i>Compatible Alternative</i>	Shared Polarity: Second Term Only Valid	Shared Polarity: First Term Only	Shared Polarity: Both Terms Compatible Initial possibility

Key: The initial possibilities considered for a conditional and a disjunction are indicated in bold font, and the first alternative in italic font. The 4 inferences with a compatible initial possibility and the 4 inferences with a compatible alternative are indicated. The shared polarity of the terms in the premise and conclusion are provided: both terms (compatible initial possibility), neither term (compatible alternative), first term only, or second term only. The 4 valid inferences are also indicated.

But the subsequent step of constructing an alternative possibility does so:

<i>If A B</i>	<i>C or D</i>	<i>(C = not-A, D = not-B)</i>
A B	C	
Not-A not-B ✓	D	
	C D ✓	

Participants will endorse the conclusion, erroneously, just as they do for inferences based on explicit negation, as Table 4 shows.

6.1. Method

6.1.1. Participants, design, procedure, and materials

The new set of 32 participants who took part in the experiment were undergraduates at the University of La Laguna. The design and procedure were the same as the previous experiment, and the materials were the same with the exception that implicit negation of terms was used instead of explicit negation.

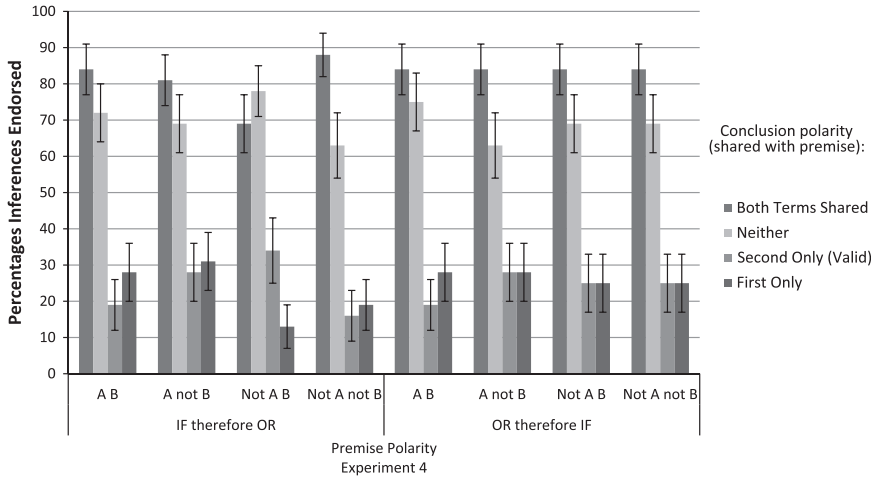


Fig. 4. The percentages of inferences endorsed between conditionals and inclusive disjunctions when the premise and conclusion share both terms (compatible initial possibility), neither (compatible alternative), the first only, or the second only, in Experiment 4 with implicit negation of terms in a binary situation. Error bars are standard error of the mean.

6.2. Results and discussion

An ANOVA of the same design as the previous experiments showed a main effect of compatibility, $F(2.188, 65.634) = 51.67$, $Mse = .16$, $p < .001$, $\eta_p^2 = .63$, due to the following differences:

- Participants endorsed more inferences that had compatible initial possibilities, i.e., the premise and conclusion shared both terms, compared to a first term only, 82% vs 25%, $F(1, 31) = 88.60$, $Mse = .06$, $p < .001$, $\eta_p^2 = .74$; or a second term only, 82% vs 24%, $F(1, 31) = 110.17$, $Mse = .05$, $p < .001$, $\eta_p^2 = .78$.
- They endorsed more inferences that had compatible alternatives, i.e., the premise and conclusion shared neither terms, compared to a first term only, 70% vs 25%, $F(1, 31) = 37.27$, $Mse = .09$, $p < .001$, $\eta_p^2 = .55$, or a second term only, 70% vs 24%, $F(1, 31) = 44.41$, $Mse = .07$, $p < .001$, $\eta_p^2 = .59$.
- The difference between inferences that had compatible initial possibilities (shared both terms) and compatible alternatives (shared neither term) was marginal, 82% vs 70%, $F(1, 31) = 3.84$, $Mse = .07$, $p = .06$, $\eta_p^2 = .11$.

There was no main effect of problem type, $F(1, 30) = .49$, $Mse = .03$, $p = .49$, $\eta_p^2 = .02$, and the two variables did not interact, $F(3, 90) = .32$, $Mse = .04$, $p = .81$, $\eta_p^2 = .01$. Participants endorsed as many inferences overall from a conditional to a disjunction as they did from a disjunction to a conditional, 49% vs 51%, as the absence of a main effect shows. They also made as many logically correct inferences from a conditional to a disjunction (yes for valid and no for invalid), as they did from a disjunction to a conditional, 38% vs 36%, $F(1, 31) = .45$, $Mse = .01$, $p = .51$, $\eta_p^2 = .02$.

A similar trend was observed for each of the inferences, in every one of the 32 observations plotted in Fig. 4, and there were no exceptions to this trend. Even for the affirmative disjunction premise, 'A or B or both' the conclusions endorsed most frequently were the compatible initial possibility 'if not-C, not-D' (84%) and equally often, the compatible alternative, 'if C, D' (75%) more so than the valid conclusion, 'if C, not-D' (19%).

The results replicate and extend those of the previous experiments for conditionals and inclusive disjunctions and show that the compatibility heuristic occurs not only with explicit negation but also with implicit negation. The results provide a definitive distinction between the compatibility heuristic and a superficial matching of the linguistic polarity of terms, since participants made more inferences

even when neither term co-referred. The next experiments further generalize it, to inferences that do not include conditionals, and inferences that do not include disjunctions.

7. Experiment 5a and 5b: Universals, disjunctions and conditionals

The aim of the experiments was to test the novel prediction that participants will rely on the compatibility heuristic even for inferences that do not depend on 'if', and inferences that do not depend on 'or', and so to test the generalizability of the heuristic to other sorts of non-categorical hypothetical inferences. We tested inferences that did not depend on 'if' by examining inferences between disjunctions and universal 'all' assertions, in Experiment 5a, and inferences that did not depend on 'or' by examining inferences between conditionals and universals in Experiment 5b. The experiments yielded similar results and so we report them together.

A universal quantifier such as 'All' can be considered equivalent in part to a conditional (e.g., [Evans et al., 1999](#); [Johnson-Laird & Byrne, 1991](#)). Logicians have developed predicate logics that express a universally quantified assertion, e.g., 'All the artists are beekeepers' by using a conditional, i.e., 'for all individuals, if the individual is an artist then the individual is a beekeeper':

$$\forall x, Ax \rightarrow Bx$$

i.e., for all x , if x is an A then x is a B (e.g. [Jeffrey, 1981](#)). When people understand a universal quantifier, e.g., 'All the artists are beekeepers' they may envisage an artist who is a beekeeper (A and B), and they may consider multiple instances of this possibility, several artists who are beekeepers ([Espino, Santamaria, Meseguer, & Carreiras, 2005](#); [Johnson-Laird & Bara, 1984](#)). If they think through all the possibilities explicitly, they will also consider individuals who are not artists but who are beekeepers (not- A and B) and individuals who are not artists and who are not beekeepers (not- A and not- B), (e.g., [Johnson-Laird & Byrne, 1991](#); [Khemlani & Johnson-Laird, 2012b](#)). The only possibility that the assertion rules out as false is an artist who is not a beekeeper (A and not- B). Hence similar possibilities are consistent with conditionals and universally quantified assertions.

An inference from a universal premise to an inclusive disjunction, e.g., 'in all gardens in which there are roses, there are lilies, therefore there are roses in the garden or there are lilies or both' (All A are B , therefore A or B or both), requires an instantiation of the premise (from 'all gardens' to 'the garden'). Granted that participants accept the instantiation, the compatibility heuristic can proceed as it does for an inference from a conditional to a disjunction. An inference from a disjunction to a universal, such as 'There are roses in the garden or there are lilies or both, therefore in all gardens in which there are roses, there are lilies' (A or B or both, therefore All A are B), requires a generalization of the premise (from 'the garden' to 'all gardens'). Once again, granted that participants accept the generalization, the compatibility heuristic can proceed as it does for an inference from a conditional to a disjunction. Generalizations of this sort are not deductively valid, although some inferences from disjunctions to universals may be inductively sound. We expect that participants will endorse inferences between universals and disjunctions when they can think of compatible initial possibilities, and compatible alternatives, even though they do not contain conditionals.

Some inferences between universals and conditionals can appear to be largely an instantiation or generalization, especially when both terms match, e.g., 'in all gardens in which there are no roses, there are lilies, therefore if there are no roses in the garden then there are lilies' (All not- A are B , therefore if not- A , B), the conclusion is an instantiation of the premise. For the inference from the conditional to the universal, e.g., 'if there are roses in the garden then there are no lilies, therefore in all gardens in which there are roses, there are no lilies' (if A not- B , therefore All A are not B), the conclusion is an induction or generalization of the premise. We expect that participants will endorse inferences between universals and conditionals, when they can think of compatible initial possibilities, and compatible alternatives, even though they do not contain disjunctions.

7.1. Method

7.1.1. Participants, design, procedure, and materials

The participants who took part in the experiment were undergraduates at the University of La Laguna. A new set of 32 participants volunteered to take part in Experiment 5a, and a different set

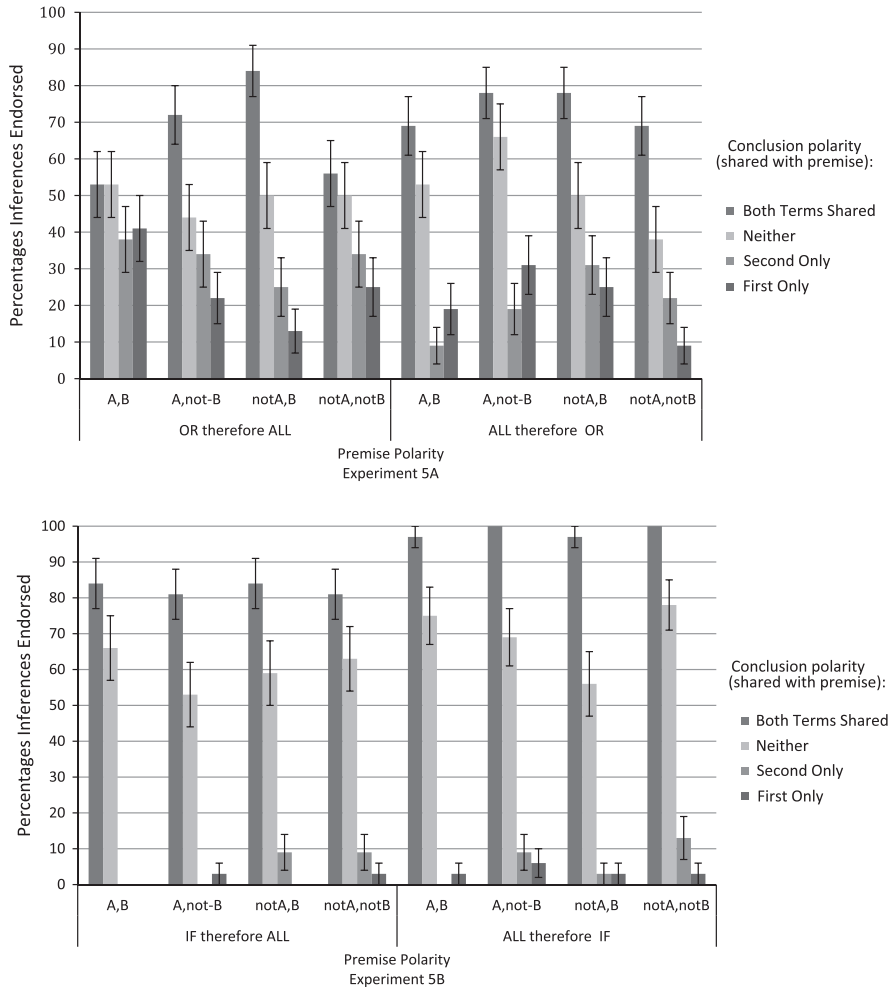


Fig. 5. (a and b) The percentages of inferences endorsed between universals and inclusive disjunctions in Experiment 5a and between universals and conditionals in Experiment 5b when the premise and conclusion share the polarity of both terms (compatible initial possibility), neither (compatible alternative), the first only, or the second only. Error bars are standard error of the mean.

of 32 participants in Experiment 5b. The design and procedure were the same as the previous experiments, and the materials were the same with the exception that universals were used such as ‘in all the zoos in which there are lions, there are tigers’, combined with disjunctions in Experiment 5a, and with conditionals in Experiment 5b.

7.2. Results and discussion

ANOVAs of the same design as the previous experiments showed that there was a main effect for compatibility, in Experiment 5a, $F(1.996, 59.890) = 26.23$, $Mse = .18$, $p < .001$, $\eta_p^2 = .47$, and in Experiment 5b, $F(1.723, 51.698) = 189.54$, $Mse = .11$, $p < .001$, $\eta_p^2 = .86$, due to the following differences:

- (a) Participants endorsed more inferences that had compatible initial possibilities, that is the premise and conclusion shared both terms, compared to a first term only, in Experiment 5a, 70% vs 23%, $F(1,31) = 40.09$, $Mse = .09$, $p < .001$, $\eta_p^2 = .56$; and in Experiment 5b, 91% vs 3%, $F(1,31) = 654.17$, $Mse = .02$, $p < .001$, $\eta_p^2 = .96$; or a second term only, in Experiment 5a, 70% vs 27%, $F(1,31) = 33.16$, $Mse = .09$, $p < .001$, $\eta_p^2 = .52$; and in Experiment 5b, 91% vs 6% $F(1,31) = 709.65$, $Mse = .02$, $p < .001$, $\eta_p^2 = .96$.
- (b) They endorsed more inferences that had compatible alternatives, i.e., the premise and conclusion shared neither term, compared to a first term only, in Experiment 5a, 50% vs 23%, $F(1,31) = 25.95$, $Mse = .05$, $p < .001$, $\eta_p^2 = .46$, and in Experiment 5b, 65% vs 3%, $F(1,31) = 125.61$, $Mse = .05$, $p < .001$, $\eta_p^2 = .80$; or a second term only, in Experiment 5a, 50% vs 27%, $F(1,31) = 14.61$, $Mse = .06$, $p < .002$, $\eta_p^2 = .32$, and in Experiment 5b, 65% vs 6%, $F(1,31) = 114.19$, $Mse = .05$, $p < .001$, $\eta_p^2 = .79$.
- (c) They endorsed more inferences for problems that had compatible initial possibilities compared to compatible alternatives, in Experiment 5a, 70% vs 51%, $F(1,31) = 9.61$, $Mse = .06$, $p < .005$, $\eta_p^2 = .24$ and in Experiment 5b, 91% vs 65%, $F(1,31) = 20.10$, $Mse = .05$, $p < .001$, $\eta_p^2 = .39$.

In Experiment 5a, there was no main effect for problem type, $F(1,30) = .42$, $Mse = .05$, $p = .52$, $\eta_p^2 = .01$, but problem type interacted with compatibility, $F(3,90) = 3.10$, $Mse = .04$, $p < .035$, $\eta_p^2 = .09$. The interaction shows that the compatibility effect occurs equally for inferences from universals to disjunctions, and for inferences from disjunctions to universals, and there is only one difference: for inferences in which the premise and conclusion shared the second term only, participants made more inferences from disjunctions to universals vs inferences from universals to disjunctions, 33% vs 20%, $F(1,31) = 6.88$, $Mse = .04$, $p < .02$, $\eta_p^2 = .18$. There is no difference for the other three cases, i.e., when they shared both terms, neither, or the first term only.

In Experiment 5b, there was a main effect for problem type, $F(1,30) = 7.70$, $Mse = .04$, $p < .01$, $\eta_p^2 = .20$, and problem type and compatibility did not interact, $F(3,90) = 2.55$, $Mse = .03$, $p = .06$, $\eta_p^2 = .08$. Participants endorsed more inferences from a universal to a conditional than they did from a conditional to a universal, 45% vs 37% as the main effect of problem type showed. The result reflects the difference between the two inferences in the strength of their conclusions (instantiation in the former and generalization in the latter).

A similar trend was observed for each polarity of premise, as Fig. 5a and b shows, and there were no exceptions, not even for the inference 'A or B, therefore All not-A are B'. The experiment shows that when participants can imagine a compatible possibility, they make an inference between a universal and a disjunction, and between a universal and a conditional. The compatibility heuristic explains the pattern of inferences, not only for inferences between conditionals and disjunctions but also for other sorts of non-categorical hypothetical inferences such as inferences based on universally quantified assertions.

8. General discussion

When someone says 'if you can't be good be careful', it may seem effortless to infer that they mean 'be good or be careful'. Yet when people cannot rely on their prior knowledge, inferences between conditionals and disjunctions represent an exceptional feat for the reasoning mind; the premise is compatible with several alternative possibilities and so too is the conclusion, and no categorical information is provided. The experiments reported here show that people make such inferences by a heuristic judgment based on whether they can think of a possibility compatible with the premise and conclusion.

8.1. The compatibility heuristic and the compatibility effect

The compatibility heuristic has three steps that are short-cuts of an algorithm for reasoning by envisaging possibilities (e.g., Johnson-Laird & Byrne, 2002). The first step is to think about the initial possibilities for the conditional and disjunction; the short-cut is that people do not think about the fully explicit possibilities for either. The second step is a compatibility test to check whether there

are any shared possibilities consistent with the conditional and the disjunction; the short-cut is that people do not carry out an incompatibility test to check there are no additional possibilities consistent with one assertion and not the other. The third step, engaged only if the first two do not deliver a compatible possibility, is to think about the most immediately available alternative, by negating both mentioned terms for the conditional; the short cut is that people flesh out just one of the alternatives for the conditional, and they do not consider alternative possibilities for the disjunction at all.

Experiment 1a and 1b showed a strong compatibility effect for inferences between 'if A B' conditionals and inclusive disjunctions. The compatibility heuristic identifies a compatible initial possibility in its first steps when the conditional and disjunction share both terms in common, and it identifies a compatible alternative in its subsequent steps when they share neither term. Participants made more inferences that shared both or neither term, compared to inferences that shared only the first or second term. Experiment 2 showed that the compatibility effect is amplified for 'A only if B' conditionals. The heuristic immediately identifies a compatible possibility in its first steps not only when the assertions share both terms but also when they share neither term.

8.2. Limitations

The limits of the compatibility heuristic are evident in inferences between bi-conditionals and exclusive disjunctions as Experiment 3 showed. The heuristic does not identify a compatible initial possibility, even when the bi-conditional and exclusive disjunction share both terms, and nor does it do so when they share neither term, the first term only or the second term only, and even after considering an alternative possibility for the bi-conditional. As a result no compatibility effect occurred for inferences between bi-conditionals and exclusive disjunctions. Instead, participants made the correct inferences more often, perhaps because the failure of the heuristic prompted them to consider explicit possibilities, and because the assertions are consistent with just two possibilities each.

The compatibility heuristic is limited to non-categorical hypothetical inferences, because it is a short-cut deployed in the absence of categorical information that would allow the elimination of possibilities. It is not deployed to reason about 'after-the-fact' hypothetical deductions, such as the affirmation of the consequent, e.g., 'if he was not good, he was careful. In fact he was careful. Therefore he was not good', because the categorical information allows participants to eliminate some possibilities from further consideration using the full algorithm.

The restriction on its deployment implies that it is not invoked simply by the complexity of a premise, e.g., for an embedded conditional such as 'If John stays sober if Mary does, the party will be a success', once it is accompanied by a categorical premise, e.g., 'The party is not a success'. The categorical information allows reasoners to eliminate some possibilities once they have unpacked the embedded conditional 'if it is the case that if Mary stays sober then John does, then the party will be a success'. They can then make the inference by constructing and eliminating models. In doing so, they are likely to encounter the usual difficulties in negating a conditional, 'it is not the case that if Mary stays sober then John does' (Espino & Byrne, 2012; Handley et al., 2006; Khemlani & Johnson-Laird, 2012a, 2012b). The inference may be a complex one but it does not invoke the compatibility heuristic.

8.3. Extensions

The compatibility heuristic may extend to further sorts of non-categorical hypothetical inference. Experiment 4 showed that it applies to inferences that do not share co-referring terms, e.g., implicit negation. The result shows that it is distinct from any superficial matching of linguistic terms. Participants did not merely make more inferences when both polarities matched, but also when neither polarity matched, and even when there was no explicit co-reference of terms. Experiment 5a and 5b showed it is not confined to inferences between conditionals and disjunctions. Participants exhibited a strong compatibility effect for inferences between universals and disjunctions, and between universals and conditionals.

A fruitful avenue for future research is to examine its role in other inferences, including those based on other connectives, e.g., 'and' and other quantifiers e.g., 'some', and the negation of assertions, e.g., 'if there is not a lion, there is a giraffe. Therefore, it is not the case that there is a lion and there is not a

giraffe.' The identification of compatible possibilities, the 'overlap' or 'intersection' of commonalities, may be a core component of reasoning (Johnson-Laird & Byrne, 2002; Sloman, 1993; Stenning & Lemon, 2001). The heuristic applies to transitive inferences from conditionals (Byrne, 1989a; Santamaria et al., 1998), e.g., 'If there's a giraffe, there's not a lion. If there's a lion, there's a hippo. Therefore if there's not a giraffe, there's a hippo'. People construct initial possibilities:

<i>If A not-B</i>	<i>if B C</i>	<i>If not-A C</i>
A not-B	B C	not-A C

They flesh out an alternative to find a compatible possibility:

<i>If A not-B</i>	<i>if B C</i>	<i>If not-A C</i>
A not-B	B C	not-A C
Not-A	B	

Hence the heuristic predicts people will make systematic errors, such as endorsing the conclusion. It may also apply to inferences such as, 'If A then B or C but not both. Either B and C or not-B and not-C but not both.' Participants construct initial possibilities:

<i>If A, B or C</i>	<i>B and C or not-</i>
	<i>B and not-C</i>
A B	B C
A C	not-B not-C

They may attempt to flesh out an alternative for the conditional to find a compatible possibility, but doing so is likely to challenge working memory capacity, since it requires negating the antecedent and consequent of the conditional, which requires negating a disjunction, a task people find difficult to accomplish (e.g., Khemlani, Orenes, & Johnson-Laird, 2012). They may reject the inference 'therefore not-A' and conclude that nothing follows.

8.4. Semantic and pragmatic factors

The compatibility heuristic has been tested for neutral content e.g., fruit, flowers, and so on in the experiments reported. Content and context affects the inferences people make, by modulating the alternatives they envisage in a process that includes the retrieval of counterexamples from knowledge, e.g., Rio is in Brazil (e.g., Johnson-Laird & Byrne, 2002). The disjunction about Obama's warning to Congress, 'Accept the tax-cut proposals or face the sequester', seems to lead to the conclusion 'if the tax-cut proposals are accepted, the sequester will not be introduced'. An inference from 'A or B' to 'if A not-B' may be considered logically invalid when 'or' is interpreted as inclusive and 'if' as conditional:

<i>A or B or both</i>	<i>if A not-B</i>
A not-B ✓	A not-B ✓
Not-A B ✓	not-A B ✓
A B x	not-A not-B x

but it is logically valid when 'or' is interpreted as exclusive and 'if' as biconditional:

<i>A or B but not both</i>	<i>if and only if A not-B</i>
A not-B ✓	A not-B ✓
Not-A B ✓	not-A B ✓

Knowledge may ensure that people interpret the disjunction and conditional in this way: they may not entertain the possibility that Congress accepts the tax-cuts proposals and the sequester is still implemented, or that Congress does not accept the tax-cuts proposals and the sequester is not implemented anyway.

Semantic and pragmatic factors affect conclusions in other non-categorical hypothetical inferences. Given, 'If John claps his hands, the bird will fly away. If the bird flies away, it is not a penguin' participants are unlikely to make the transitive inference 'If John claps his hands, the bird is not a penguin'. The conclusion combines a causal relation from the first conditional (John clapping his hands causes the bird to fly away) and a definitional relation from the second (a bird that flies is by definition not a penguin), and requires a reasoner to accept that John's clapping causes the bird to be not a penguin. But people can access counterexamples to this conclusion from their knowledge, e.g., John clapping his hands and the bird being a penguin, and so the causal antecedent of the first conditional is not inherited as a cause for the definitional consequent of the second.

The compatibility heuristic is based on a short-cut for considering alternative possibilities; it may be interesting to consider whether a short-cut based on considering probabilities could also be devised. According to the probability account content and context affect reasoning by influencing the assessment of the likelihood of the individual components described by the conditional and their relation based on prior beliefs (e.g., [Evans & Over, 2004](#)). It may also be interesting to examine whether the compatibility effect occurs when participants are required to make other judgments, e.g., about likelihood (e.g., [Oberauer et al., 2011](#); [Over et al., 2010](#)), given that counterexamples have different effects depending on whether participants make an inference or a likelihood judgment ([Geiger & Oberauer, 2007](#); [Markovits et al., 2010](#)).

9. Conclusions

Simple inferences are firmly grounded in reality, but everyday mental life is not: people often base their inferences not on categorical facts, but on non-categorical hypothetical suppositions (e.g., [Wason & Johnson-Laird, 1972](#)). Such inferences free the reasoning mind from the shackles of facts, to support purely suppositional thought. It is a pinnacle of achievement to entertain such genuinely hypothetical deductions. They allow people to leap from one hypothetical possibility to another, without having to set foot on the ground of categorical facts. However, the experiments reported here suggest the mental reality may be somewhat more prosaic. Inferences between conditionals and disjunctions are based on an immediate heuristic: people make their judgment by envisaging initial possibilities, rather than fully thinking through all the possibilities consistent with the conditional and disjunction. They assess compatibility, rather than checking for incompatible possibilities, and so make a judgment of possibility rather than necessity. They appear to make little attempt to evaluate the deductive validity of the inference, because of the difficulty of considering multiple alternatives in the absence of any facts. They seem unable to resist the temptation of the simplicity of a single state of affairs: their immediate intuition is to accept an inference when the premise and conclusion share a compatible possibility.

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