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### It is not the case that if you understand a conditional you know how to negate it

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# It is not the case that if you understand a conditional you know how to negate it

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We report the results of an experiment to test the cognitive processes underlying inferences from negated conditionals, e.g., ‘it is not the case that if A then B’. The results show that people endorse not only the conditional conclusion ‘if A then not B’ but also ‘if not A then B’. We discuss the implications of the results for competing explanations of reasoning about negated conditionals.

**Keywords:** Conditionals; Negation; Reasoning.

## INTRODUCTION

People deny each other’s assertions in many situations, from everyday conversation to political and scientific discourse (e.g., Wason, 1965), and they revise their own beliefs in the light of inconsistencies and contradictions (Johnson-Laird, Girotto, & Legrenzi, 2004). But they can find it difficult to work out what follows from the denial of a compound assertion (Byrne & Handley, 1992). Suppose Alicia says: ‘if you understand a conditional, you know how to negate it’. Mark says: ‘No! It isn’t the case that if you understand a conditional you know how to negate it’. What conclusion follows from his assertion? Does it follow that ‘you understand a conditional and do not know how to negate it’? Or does it follow that ‘if you understand a conditional, you do not know how to negate it’? In fact, the data show that people tend to accept both conclusions frequently (Handley, Evans, & Thompson, 2006). But are there any compound conclusions they are *not* willing to accept? For example, does it follow that, ‘if you do not

understand a conditional, you know how to negate it’? The answer is not known. Yet it is crucial, not only for understanding how people refute each other’s hypotheses, change their plans, or modify their causal ascriptions, but also for distinguishing between competing explanations of the cognitive processes underlying conditional reasoning and negation.

One explanation for how people deny a conditional such as ‘it is not the case that if A then B’, is that they imagine the true possibilities for the conditional and then construct their complement (Khemlani, Orenes, & Johnson-Laird, 2010). People construct an initial mental model consisting of the elements mentioned in the conditional, ‘A and B’. They can flesh out an explicit set of models that includes other true possibilities: ‘A and B’, ‘not-A and not-B’ and ‘not-A and B’ as Table 1 shows (Johnson-Laird & Byrne, 2002). The theory predicts a single correct response: people accept the correct conclusion ‘A and not-B’ when they accurately negate the conditional by constructing the complement of the explicit set of models, as Table 2 shows. However, people err in

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negating the conditional by constructing the complement of the initial mental models ‘A and B’, which is: ‘A and not-B’, ‘not-A and B’, and ‘not-A and not-B’. The wide scope of the sentential operator ‘it is not the case’ is thus erroneously restricted to the conditional’s components (Barres & Johnson-Laird, 2003). As a result, the theory predicts a set of incorrect responses that arise from this error. One incorrect response is that people accept, ‘if A then not-B’ because they construct its initial mental model, ‘A and not-B’ (see Table 1) and establish that there is a consistent possibility in the complement of the initial mental models of the conditional and in the mental model of this conclusion (see Table 2). But the model theory predicts other conditional errors: participants who err in negating the conditional should accept not only the conclusion ‘if A then not-B’ but also ‘if not-A then B’, and even ‘if not-A then not-B’ because the initial mental model for each of them (respectively, not-A and B; not-A and not-B) is also represented as a consistent possibility in the complement of the initial mental models of the conditional (see Table 2). Of course the theory predicts that participants should reject some conclusions, e.g., ‘if A then B’ from the negation of the conditional, even if they have erred in negating the conditional. These predictions are readily falsified, e.g., if people reject ‘if not-A then B’ or endorse ‘if A then B’.

A different explanation of the negation of a conditional is that people add the ‘if’ clause of the conditional (A) to their beliefs and assess whether

**TABLE 1**

The initial and explicit models for four sorts of conditionals according to the mental model theory (e.g., Johnson-Laird & Byrne, 2002)

	<i>Initial models</i>		<i>Explicit models</i>	
Conditional				
If A then B	A	B	A not-A not-A	B not-B B
If A then not B	A	not-B	A not-A not-A	not-B B not-B
If not-A then B	not-A	B	not-A A A	B not-B B
If not-A then not-B	not-A	not-B	not-A A A	not-B B not-B

**TABLE 2**

The negation of a conditional ‘it is not the case that if A then B’ based on the construction of the complement of the initial mental models of ‘if A then B’ and the explicit models of ‘if A then B’

	<i>Models of if A then B</i>		<i>Complement</i>	
Initial models	A	B	A not-A not-A	not-B not-B B
Explicit models	A not-A not-A	B not-B B	A	not-B

or not the ‘then’ clause follows (B or not-B) (Evans & Over, 2004). According to this ‘suppositional conditional’ account, the negated conditional, ‘it is not the case that if there is an A then there is a B’, is understood by inferring, correctly on this account, ‘if there is an A then there is not a B’. When participants are given a negated conditional, they endorse the conclusion ‘if A then not-B’ (Handley et al., 2006). The result has been claimed as strong support, in fact, as a ‘litmus test’, of the suppositional conditional theory’s tenet that there is a special correspondence between ‘it is not the case that if A then B’ and ‘if A then not-B’. The theory also predicts that certain conclusions should not be endorsed: “it follows that people should be unwilling to infer that p follows, not-q follows or the conjunction of p and not-q follows from the negation of a conditional” (Handley et al., 2006, p. 566). The finding that participants do, in fact, judge that the conjunction of ‘p and not q’ follows from the negation of a conditional has been described as an error on this view. The theory proposes that it arises because a simulation of ‘A and not-B’ is consistent with the conditional’s negation (Handley et al., 2006). On this account, people should not endorse conclusions such as ‘if not-A then B’ because they think about true antecedents only (A) and so they do not think about their belief in the negated ‘if’ clause (not-A) or whether or not the ‘then’ clause follows in such circumstances (Handley et al., 2006; Over, Hadjichristidis, Evans, Handley, & Sloman, 2007). Given that people do not think about the ‘not-A’ instance, the account predicts that participants will reject ‘if not-A then B’ and ‘if not-A then not-B’, rather than accept them.

When participants have been given a negated conditional ‘it is not the case that if A then B’ and then given just two compound conclusions,

'A and not-B' and 'if A then not-B', most participants judge that 'A and not-B' follows, as the model theory predicts; and most participants also judge that 'if A then not-B' follows, as the suppositional conditional account predicts (Handley et al., 2006, Experiments 1 and 2). But it is not yet known how participants respond when they are asked whether other compound conclusions follow, e.g., 'if not-A then B'. And it is their responses to these other compound conclusions that can help to distinguish between the alternative theories. We report the results of an experiment that tests the different predictions from the alternative theories by examining the inferences people make from negated conditionals when they are given the full set of compound conclusions.

## EXPERIMENT

The experiment examined the full set of conditional and conjunctive conclusions that people judge to follow from a negated conditional, 'it is not the case that if A then B'. We used the same task and instructions as reported in Handley et al. (2006), but we examined all four possible conditional conclusions ('if A then B', 'if A then not-B', 'if not-A then B', and 'if not-A then not-B'). We also examined all four possible conjunctive conclusions ('A and B', 'A and not-B', 'not-A and B', 'not-A and not-B').

According to the mental model theory, if participants compare a mental model of each conclusion to the complement of the *explicit* models of the negated conditional, then they will endorse 'A and not-B'. If they compare a mental model of each conclusion to the complement of the *mental* models of the negated conditional (rather than the explicit models), they will make the mistake of endorsing 'not-A and B' and 'not-A and not-B'. They will also endorse all three conditional conclusions 'if A then not-B', 'if not-A then B' and 'if not-A then not-B' (Byrne & Johnson-Laird, 2009). They will reject 'if A then B' and 'A and B'. According to the suppositional conditional account, they will accept 'if A then not-B' but reject 'if not-A then B' and 'if not-A then not-B', because the former corresponds uniquely to their interpretation of a suppositional conditional. Similarly they will accept only 'A and not-B'-an error arising from the simulation of A-but reject 'not-A and B' and

'not-A and not-B', given that the negated antecedent is not considered (Handley et al, 2006).

## Method

### *Participants*

The participants were 92 undergraduate students from the University of La Laguna, Tenerife, Spain. The participants were Spanish and they completed the tasks in their native Spanish.

### *Design, materials and procedure*

Each participant acted as his or her own control and the design was a within-participants one. Each problem consisted of a negated conditional, 'it is not the case that if A then B' and a conclusion which was either a conditional or a conjunction. Each participant completed problems for the two types of conclusions. Participants received a booklet which contained the following instructions on the first page:

'This task is designed to test your understanding of logical rules. On the following pages you will be presented with a series of problems. In each problem, a rule will be presented followed by a conclusion. For each problem you must indicate whether or not the conclusion necessarily follows given the rule that precedes it. If you think that the conclusion follows the rule you should mark the option 'yes', if you think that the conclusion does not follow the rule you should mark the option 'no'. A conclusion is necessarily true when the conclusion must follow given the truth of the rule.'

There were 24 problems and a single problem was presented on each of the remaining 24 pages in the booklet. There were four conditional conclusions (if A then B, if A then not-B, if not-A then B, and if not-A then not-B) and four conjunction conclusions (A and B, A and not-B, not-A and B, not-A and not-B). Each problem was instantiated in three different contents – letters and numbers (i.e., A and 3), fruits and animals (i.e., oranges and gorillas) and flowers and birds (i.e., roses and pigeons), that is, 24 problems in total. Each problem was embedded in the context of a computer printing out cards. An example of a problem is as follows:

A computer has printed a card with a flower on the left side and a bird on the right side. The computer followed the rule:

It is not the case that if the flower was a rose then the bird was a pigeon.

Does it follow that:

The flower printed was a rose and the bird printed was not a pigeon.

YES NO

Participants were asked to indicate their response by circling one of the options for each problem.

### Results

Participants frequently endorsed the conclusions ‘if not-A then B’, and ‘if A then not-B’ as well as ‘not-A and B’ and ‘A and not-B’, as shown by an ANOVA on the ‘yes’ responses, in a repeated measures 2 (conclusion connective: if, and) by 4 (conclusion polarity: A B, A not-B, not-A B, not-A not-B) design using a Greenhouse–Geisser correction for the violation of the sphericity assumption. There was no main effect of conclusion connective,  $F(1, 91) = 1.60, MSe = .050, p = .20$ ; a main effect of conclusion polarity,  $F(3, 273) = 14.27, MSe = .31, p < .001, \eta_p^2 = .13$ , and the two variables interacted,  $F(2.823, 256.8) = 8.41, MSe = .047, p < .001, \eta_p^2 = .085$ .

Planned comparisons with a Bonferroni correction showed that ‘if A then not-B’ (64%) and ‘if not-A then B’ (63%) were endorsed equally often,  $t(91) = .28, p > .05, \eta_p^2 = .001$ , as Table 3 shows. However, ‘if A then not-B’ was endorsed more often than ‘if not-A then not-B’ (38%),  $t(91) = 3.27, p < .01, \eta_p^2 = .13$ . It was, of course, endorsed more often than ‘if A then B’ (27%),  $t(91) = 5.22, p < .01, \eta_p^2 = .23$ .

Likewise, ‘A and not-B’ (57%) and ‘not-A and B’ (61%) were endorsed equally often,  $t(91) = 1.04, p > .05, \eta_p^2 = .012$ . ‘A and not-B’ and

‘not-A and not-B’ were also endorsed equally often (51%),  $t(91) = .86, p > .05, \eta_p^2 = .008$ ; ‘A and not-B’ was, of course, endorsed more often than ‘A and B’ (32%),  $t(91) = 3.74, p < .01, \eta_p^2 = .13$ .

### DISCUSSION

When participants understand the negation of a conditional ‘it is not the case that if A then B’ they endorse not only ‘if A then not-B’, but also ‘if not-A then B’. It is difficult to construe this finding as compatible with the idea that there is a single special correspondence between ‘if A then not-B’ and a specific interpretation of the conditional, as proposed by the suppositional account. In previous experiments, when participants were given a negated conditional, they endorsed the only conditional conclusion provided to them, ‘if A then not-B’, and it was concluded that there is a special relationship between ‘it is not the case that if A then B’ and ‘if A then not-B’ (Handley et al., 2006). But we have examined the full set of conditional conclusions, including ‘if not-A then B’ and we can conclude that there is no special relationship between a negated conditional and ‘if A then not-B’, given that participants endorse other conditionals too, e.g., ‘if not-A then B’.

The result was predicted by the view that participants construct a *mental* model of the conditional, ‘A and B’, rather than an *explicit* set of models, and they represent its negation by constructing the complement of these mental models, hence restricting the wide-scope of the negation to the clauses of the conditional (Byrne & Johnson-Laird, 2009). Participants endorse both of the conditionals with a negated component because the mental model of each one can be matched to a model in the erroneous complement of the mental models of the conditional, as Table 1 shows. However, participants did not endorse ‘if not-A then not-B’ as predicted by this account, perhaps indicating that they did not construct the complete complement of their mental models. Participants who construct mental models of the conditional, i.e., ‘A and B’, may find it effortful to construct the fully explicit models of the complement, and may instead construct mental models of the complement, i.e., ‘A and not-B, not-A and B’.

The results also show that participants endorse each of the conjunction conclusions with a

TABLE 3

Percentages of ‘yes’ responses to the different types of conclusions based on ‘if’ and ‘and’ for a negated conditional ‘it is not the case that if A then B’ in response to the question ‘does it follow that ...’

Conclusion components	not-A			
	A B	A not-B	not-A B	not-B
Conclusion connective				
If	27	64	63	38
And	32	57	61	51

negated component. It is difficult to construe the pattern of endorsements of conjunction conclusions as compatible with the idea that participants think about only 'A and not-B' in a suppositional chain when they understand the conditional, and that they do not think about 'not-A and B' and 'not-A and not-B' (pace Handley et al., 2006). The results were predicted by the mental model theory: the response 'A and not-B' corresponds to the correct conclusion constructed from the complement of the explicit models, whereas the other two responses, 'not-A and B' and 'not-A and not-B' are errors arising from the complement of the mental models. One anomalous finding is that participants on one third of trials judged that 'A and B' follows; the result may reflect the confusion engendered by negated conditionals. It may be useful to compare negated and affirmed conditionals to examine the tendency to endorse the various conclusions in the full set of compound conclusions.

A potential alternative explanation is that participants' inference of the conjunctions indicates that they have misinterpreted the conditional as a 'biconditional'. If participants misinterpret the negated conditional as a negated biconditional, they could construct explicit models that rule out as impossible not only 'A and not-B' but also 'not-A and B' according to the model theory (Johnson-Laird & Byrne, 2002); or they could consider suppositional chains of 'A and not-B', and 'B and not-A' if their interpretation is equivalent to 'if A then B and if B then A' according to the suppositional account (Evans et al., 2003). However, the biconditional explanation is at best a partial one, since participants also accept 'not-A and not-B'.

People find it hard to understand accurately the denial of a compound assertion. They are capable of doing so however, as evidenced by their robust rejection of the compound assertion itself following its denial. But they do not reject either of the conditional conclusions that contain a single negative component. The experiment shows that their understanding of 'it is not the case that if A then B' allows them to conclude not only 'if A then not-B' but also 'if not-A then B'. We have also found similar results using a task that allows not only the 'yes' and 'no' responses used in previous studies of negated conditionals

(Handley et al., 2006), but includes a third 'indeterminate' choice.

Negation is a central logical and psychological concept and people can communicate their denial and rejection of assertions and beliefs effectively (Espino, Santamaria, & Byrne, 2009). However, they take short-cuts to do so, and one common short-cut is to envisage a single true possibility for an assertion and to understand its denial by envisaging the complement of these mental models. The short-cuts result in errors in the application of sentential operators such as negation, which are more wide-spread than previous research has indicated.

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